Heterogeneity and Trade

Consequences for Gravity, Home Market Effect and Price Indices

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Preface

Takk til rettleiar Alfonso Irarrazabal som ser resultat der eg ikkje ser det, og som har funne særs interessante artiklar for meg å jobba med. Eg har lært mykje meir av denne prosessen enn det som til slutt er med i oppgåva.

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Takk til instituttet som har gjeve meg mange fine år.

Takk til kona som er so grei. Ingenting hadde vore det same utan deg.

Contents

1. Abstract

This thesis analyses in detail two models of international trade featuring monopolistic competition and increasing returns. The first model is a standard Krugman model, while the other is my adaptation of a model due to Thomas Chaney (2008). Both models include one homogeneous industry, with constant returns and perfect competition, in addition to one differentiated sector with increasing returns. The models build on the Dixit-Stiglitz framework, and in my version both have only two countries.

The difference between the models is that Chaney introduces heterogeneous productivity in the differentiated sector, and has a system of fixed costs for entry into each market. Thus a firm may well compete in one market but not in the other.

This new specification of the model leads to interesting findings. Chaney himself analyses the consequences for gravity. I follow this up by investigating how his findings carry over to the two country case. My findings are that his results hold also in this case. For the n-country case, the Krugman model implies that the elasticity of substitution (σ) affects the elasticity of trade flows with respect to variable trade costs (ς) positively. Chaney finds that this effect is absent in his model. On the contrary, the elasticity of trade flows with respect to fixed trade costs (ξ) is affected negatively by σ. Thus the sum of this is that trade flows are affected negatively by σ, not positively as Krugman's model implies.

In the two-country case, the results are stronger. Here the Krugman model implies a positive relationship between σ and ς while the Chaney-model implies a negative. In addition, the effect on ξ is negative, and more so than in the n-country case.

My next results come from analysing the home market effect (HME) in the model. I find that the effect is present, but is graphically visible only for the right parameter values. It thus seems to be rather week. The result I obtain is a non-linear relationship between the share of labour force and the share of production. In the case of

symmetric costs and equal wages, the smaller country will produce a somewhat less than proportional share of the differentiated good.

The non-linear shape still ensures that we never get full specialisation. The smallest countries will always produce some of the differentiated good. This is caused by the productivity structure: there will always be a few firms productive enough to overcome the disadvantages of being located in the smaller market.

Finally, I analyse the impact of a change in country size on the ideal price indices. As income per capita is constant, the impact on the price index can be seen as a proxy for the impact on utility. I derive how the gains from trade (as measured by the price index) depend on the size of the partner one trades with.

For the Krugman model I find that a country benefits from getting bigger. Somewhat surprisingly I find that if a homogeneous good is also traded, it matters not for a country whether the trading partner is small or large. For the Chaney model the results are less clear, but a combination of algebra and numerical analysis using MATLAB indicates that a country gains from getting bigger, and that it also gains from having a larger trading partner. The results are not entirely clear on which country benefits the most in absolute terms, but it seems clear that the growing country will a larger percentagewise reduction in the price index.

2. Introduction

The Home Market Effect (HME) is a well documentet phenomena that countries tend to export goods that they have a relatively large demand of. An alternative formulation that will be more relevant for this thesis is that everything else equal the larger (of two) country will be responsible for a larger than proportional share of world production of increasing return goods. This is contrary to traditional trade models where countries specialise according the comparative advantage, but finds theoretical support in the works of among others Paul Krugman. The specific source I will use for this model is section 10.4 in Helpman and Krugman (1985).

By only changing the model in one point: introducing trade cost also on the homogeneous good, Davis (1998) shows that the HME disappears entirely, unless differences in trade costs are large. It has subsequently been shown (Holmes, Stevens, 2002) that the results can be re-obtained by leaving the two-sector case and introducing a range of industries differentiated by the degree of scale economies. To do this they are forced to step out of the Dixit-Stiglitz model of monopolistic competition. They call this to *breathe new life into Krugman's original findings*, but I cannot see that much is left of the Krugman model at all.

Feenstra, Markusen and Rose (1998) show that the HME can also be found in a model with homogeneous goods with restricted entry. All in all, it seems that the HME is a general phenomenon in the theory, but that it rests heavily on a range of parameters.

In this thesis I will not deviate as much from the Krugman framework as those above. I will follow the model of Thomas Chaney (2008) where he introduces heterogeneous productivity and a new system of fixed costs, where each firm must pay a fixed cost for entry into each market.

The structure of the thesis is as follows. In *chapter 3* I go through the Krugman framework that I will use for comparison. This section builds on Krugman 1980 and Helpman/Krugman 1985. I deviate somewhat in the structure of their presentation and show more derivations than they do. This does not contain much of an independent contribution. In *chapter 4* I turn to Chaney's model. I go through the derivations of his model step by step, and again, show more derivations. Chaney is concerned with the gravity effect. In *chapters 5, 6* and *7* my independent contributions can be found.

1) In *chapter 5* I present Chaney's finding on the gravity properties of the model. I translate the results into a two-country case for better comparison to Helpman/Krugman's results.

2) In *chapter 6* I investigate whether the home market effect exists in the Chaney model and if so, how the effect deviates from Krugman's result.

3) In *chapter 7* I look at the affect of a change in relative size of countries on the ideal price indices of the two countries, and compare that to what can be derived for the Krugman framework. I compare two models with exogenous wages which means that this investigation is really an investigation of the gains from trade and how this is dependent on relative size.

I focus a lot on relative size in this thesis. When I study the impact of *a change in relative size* or a *growth* in the size of one country, that is not because I am interested in the dynamics of growth in itself. It is just a way of expressing that I change the assumptions about relative size to see how this affects the results.

3. The Krugman Framework

3.1 Deriving the basics

In this section I will present a basic model of trade in differentiated goods. The basic framework of monopolistic competition and economies of scale in the first part is more or less taken from Krugman (1980), and the second part that is specifically on the home market effect (HME) is taken from Helpman/Krugman (H/K) (1985). The first of these sections can be taken lightly on, but some references to this section will come later. It is the section on the home market effect that will be the foundation for most of the comparison.

I have changed a little from their setup and I have chosen to present it in a notation closer to that in the article by Thomas Chaney. This notation is easier to adapt to a multi-country case or other generalisations.

This world consists of two countries where productivity is homogeneous. Countries only differ in size, measured in labour available. The model has one factor of production: labour L.

The representative consumers' utility is given by:

$$u = \sum_i c_i^{(\sigma-1)/\sigma}, \qquad 1 < \sigma \tag{1}$$

with budget constraint $\sum_i p_i c_i \leq w$. σ denotes the elasticity of substitution between varieties. This is identical for both countries. We have a production function with one factor of production:

$$l_i = \alpha + \beta x_i, \qquad \alpha, \beta > 0, i = 1, \dots, n \tag{2}$$

Output equals demand:

$$x_i = L_K c_i, \qquad i = 1, \dots, n \tag{3}$$

Where L_K means population in country *K*.

Full employment condition:

$$L_K = \sum_{i=1}^{n_K} (\alpha + \beta x_i) \ , \qquad K = H, F \tag{4}$$

The optimisation problem gives the Lagrangian:

$$\mathcal{L} = \sum_i c_i^{(\sigma-1)/\sigma} - \lambda \left(\sum_i p_i c_i - w \right)$$

$$\frac{\partial \mathcal{L}}{\partial c_i} = \frac{\sigma - 1}{\sigma} c_i^{-1/\sigma} - \ \lambda p_i \rightarrow \frac{\sigma - 1}{\sigma} c_i^{-1/\sigma} = \lambda p_i \ , \qquad i = 1, \ldots., n \quad (5)$$

(In the Lagrangian above, we have replaced utility with a strictly increasing transformation that will yield the same results.)

We solve (5) for c_i

$$c_i = \left(\frac{\lambda p_i \sigma}{\sigma - 1} \right)^{-\sigma}$$

We go through the same procedure for a second variety *j*, and divide the one by the other:

$$\frac{c_i}{c_j} = \left(\frac{p_i}{p_j} \right)^{-\sigma}$$

multiplying both sides by $p_i c_j$ and summing over *i* on both sides

$$\sum_i p_i c_i = p_j{}^{\sigma} c_j \sum_i p_i{}^{1-\sigma}$$

Now the left hand side is equal to income *w*. To get the (Marshallian) demand function we must isolate c_j and at last define the price index for country *K* as

$$\mathrm{P} \equiv \left(\sum_i p_i{}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

to get

$$c_j = \frac{p_j{}^{-\sigma}}{\mathrm{P}^{1-\sigma}} w$$

On the aggregate where supply equals demand:

$$x_i = L_K c_j = \frac{p_j{}^{-\sigma}}{\mathrm{P}^{1-\sigma}} w L_K \qquad (6)$$

Assuming that there is a considerable number of firms, the price of an individual good does not affect the marginal utility of income. The elasticity of substitution is therefore $-\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = \sigma$

Krugman utilises the approach of Chamberlinian imperfect competition: Firms maximise profits, free entry drives profits to zero (assuming $\alpha > 0$). Optimal price for the firms will be a constant markup over marginal cost, just covering fixed costs:

$$\pi_i = p_i x_i - (\alpha + \beta x_i)w, \quad i = 1, \ldots, n \qquad (7)$$

$$p_i = \frac{\sigma}{\sigma - 1}\beta w, \qquad i = 1, \ldots, n \qquad (8)$$

As σ, β and *w* are common to all firms, p and π are also the same. We can skip the subscript denoting firm.

We use the zero profit assumption $\pi = 0$ and solve for x_i:

$$x_i = \frac{\alpha w}{p - \beta w}$$

We solve (7) for *w* and insert it above to get

$$x_i = \frac{\alpha(\sigma - 1)}{\beta}, \quad i = 1, \ldots, n \qquad (9)$$

which is independent of the size of the country, and therefore equal to all countries. It is also independent of trade. Trade therefore realises no gains due to scale of production. We can see that as α, β and θ are equal to all firms, we can also on *x* skip subscript.

The number of varieties produced can be determined from full employment condition (4) and (9).

$$L_K = n_K(\alpha + \beta x)\,, \qquad K = H, F$$

Solving for n_K and simplifying

$$n_K = \frac{L_K}{\sigma\alpha}, \qquad K = H, F \tag{10}$$

where n_K is the number of varieties produced in country *K*. This is independent of trade. We see that that in this model gains from trade only comes through one channel: an increase in available goods for consumers. Even with unchanged real wages *w/p*, we have an increase in utility.[1]

3.2 Trade flows

Individuals in country *H* will spend a fraction $\frac{n_F}{n_H+n_F}$ on foreign goods. Foreigners will spend $\frac{n_H}{n_H+n_F}$ on *home* goods. The value of home country imports measured in wage units is $L_H\frac{n_F}{n_H+n_F}$ which we from (10) can see equals $\frac{L_H L_F}{L_H+L_F}$. We get the same result when finding the value of foreign country imports, which implies trade balance.
We have seen that the volume of trade is determinate, but the direction is indeterminate.

[1] A short note that shows this is included as the appendix *gains from variety*

3.3 Transport costs

We now introduce transport costs of the iceberg type. Of the shipped goods, only a fraction g arrives, $1 - g$ is lost in transit. There are no other costs attached to the involvement to trade than this variable unit cost.

With the introduction of trade costs, identical prices across countries are no longer guaranteed. We now introduce different prices across countries:

- In country H, home goods (n_H varieties) are sold at f.o.b. price p_H.
- In country F, foreign goods (n_F varieties) are sold at f.o.b. price p_F.
- Foreign goods are sold in H at c.i.f. price $p_{FH} = \frac{p_F}{g}$.
- Home goods are sold in F at c.i.f. price $p_{HF} = \frac{p_H}{g}$.

We now get that consumers will consume unequal amounts of the goods. It can be shown by maximising H-consumers' utility with respect to H-goods and F-goods, dividing the one by the other and solving for the consumption of F-good c_{FH} that $c_{FH} = \left(\frac{p_H}{p_{FH}}\right)^{\sigma} c_H$, where c_H is the individual consumption of a single variety of an H-good in country *H*.

Due to the transport costs, one unit of an imported good consumed, means that 1/g units must be produced.

I define a variable θ_K as *the ratio of total demand (reaching producer) by K-residents for each imported variety to demand for each local variety.* We must take account of the goods lost in transport.

$$\theta_K = \frac{1}{g}\frac{c_{LK}}{c_K}, \qquad K, L = H, F \tag{11}$$

Inserting for c_{FH} and c_{HF} from above:

$$\theta_{\mathrm{H}} = \frac{1}{\mathrm{g}}\left(\frac{p_H}{p_{FH}}\right)^{\sigma} = \frac{1}{\mathrm{g}}\left(\frac{p_H}{p_F/g}\right)^{\sigma} = \left(\frac{p_H}{p_F}\right)^{\sigma} g^{\sigma-1}$$

$$\theta_{\mathrm{F}} = \frac{1}{\mathrm{g}}\left(\frac{p_F}{p_{HF}}\right)^{\sigma} = \frac{1}{\mathrm{g}}\left(\frac{p_F}{p_H/g}\right)^{\sigma} = \left(\frac{p_H}{p_F}\right)^{-\sigma} g^{\sigma-1} \qquad (12)$$

We know that spending must be equal to wages (which are now allowed to differ between countries). For country K we have the condition $(n_K p_K + \theta_{\mathrm{K}} n_L p_L)c_K = w_K$. We can also find the elasticity of *export* demand and *domestic* demand facing each firm:

$$c_{KL} = \left(\frac{p_L}{p_{KL}}\right)^{\sigma} c_L = g^{\sigma}\left(\frac{p_L}{p_K}\right)^{\sigma} c_L\,, \qquad K, L = H, F$$

$$-\frac{\partial c_K}{\partial p_K}\frac{p_K}{c_K} = \sigma$$

$$-\frac{\partial c_{KL}}{\partial p_K}\frac{p_K}{c_{KL}} = \sigma$$

The elasticity facing a given firm is the same, whether it comes from abroad or the same country. We bring back (7) and (10)

$$p_K = \frac{\sigma}{\sigma-1}\beta w_K, \qquad i = 1, \dots., n$$

$$n_K = \frac{L_K}{\sigma\alpha}, \qquad K = H, F \qquad (13)$$

As can be seen from (13), prices can only differ if wages differ, something they are now (with transport costs) allowed to do. Prices relative to wages are constant. The number of varieties produced is unchanged, trade or not, trade costs or not. The only thing that can change is relative wages $\frac{w_H}{w_F} = \omega$.

This is the framework we need. we will do certain changes when we go to the next subsection, but the changes are rather trivial, and we do not have to derive everything again.

3.4 Home Market Effects on the Pattern of Trade

Krugman's main result is that countries will tend to specialise in production of goods that they have a relatively large demand of. In this section I will leave the 1980 article and more or less follow the model in section 10.4 of Helpman and Krugman (1985) (may be referred to as H/K from now).

We let c_0 denote the consumption of the homogeneous good. Utility is now given by:

$$U = {c_0}^{1-\mu}\left(\sum_{i=1}^{n_H} c_i^{(\sigma-1)/\sigma} + \sum_{j=1}^{n_F} c_j^{(\sigma-1)/\sigma}\right)^{\frac{\sigma\mu}{\sigma-1}}, \qquad 1 < \sigma, \qquad 0 < \mu < 1 \qquad (14a)$$

subject to the budget for H:

$$p_0 c_0 + \sum_{i=1}^{n_H} p_i c_i + \sum_{j=1}^{n_F} \tau p_j\, c_j \leq w \qquad (15a)$$

budget for F:

$$p_0 c_0 + \sum_{i=1}^{n_H} \tau p_i c_i + \sum_{j=1}^{n_F} p_j\, c_j \leq w \qquad (15a)$$

Where τ denotes transport costs between the countries. Costs are assumed identical in both directions. Transport costs are of the iceberg type. If one unit of good is to be imported, τ units must be exported from the other country.

Wages are identical in both countries. This is because we assume both countries produce the homogeneous good with the same technology and, most crucially, the homogeneous good is assumed free of trade costs.

This means all prices f.o.b. are the same, as given by (7), $p_H = p_F = p$. I will still keep the subscripts for now.

We simplify the utility function and the constraints as we have symmetry between varieties:

$$U = {c_0}^{1-\mu}\left(n_H c_H^{(\sigma-1)/\sigma} + n_F c_{FH}^{(\sigma-1)/\sigma}\right)^{\frac{\sigma\mu}{\sigma-1}}, \qquad 1 < \sigma \qquad (14b)$$

subject to the budget for H:

$$p_0 c_0 + n_H p_H c_H + n_F \tau p_F c_F \leq w \qquad (15b)$$

budget for F:

$$p_0 c_0 + n_H \tau p_H c_H + n_F p_F c_F \leq w \qquad (15b)$$

As we are mostly interested in the differentiated good, and a constant share of income will be spent on homogeneous goods, we can alternatively express the budget constraints as

$$n_H p_H c_H + n_F \tau p_F c_{FH} \leq \mu w \qquad (15c)$$

$$n_H \tau p_H c_{HF} + n_F p_F c_F \leq \mu w \qquad (15c)$$

Due to CD-utility c_0 can easily be found as $c_0 = (1-\mu)\frac{w}{p_0}$

Given this, we can consider c_0 to be exogenous to the sub-optimisation between imported and domestically produced goods, and use the constraints in (15c)

For country H we get the Lagrangian

$$\mathcal{L}_H = {c_0}^{1-\mu}\left(n_H c_H^{(\sigma-1)/\sigma} + n_F c_{FH}^{(\sigma-1)/\sigma}\right)^{\frac{\sigma\mu}{\sigma-1}} - \lambda(n_H p_H c_H + n_F \tau p_F c_F \leq -\mu w)$$

$$\frac{\partial \mathcal{L}_H}{\partial c_H} = {c_0}^{1-\mu}\frac{\sigma\mu}{\sigma-1}\left(n_H c_H^{(\sigma-1)/\sigma} + n_F c_{FH}^{(\sigma-1)/\sigma}\right)^{\frac{\sigma\mu}{\sigma-1}-1} \times \frac{\sigma-1}{\sigma} n_H c_H^{-1/\sigma} - \lambda n_H p_H = 0$$

$$\frac{\partial \mathcal{L}_H}{\partial c_F} = {c_0}^{1-\mu}\frac{\sigma\mu}{\sigma-1}\left(n_H c_H^{(\sigma-1)/\sigma} + n_F c_{FH}^{(\sigma-1)/\sigma}\right)^{\frac{\sigma\mu}{\sigma-1}-1} \times \frac{\sigma-1}{\sigma} n_F c_{FH}^{-1/\sigma} - \lambda n_F \tau p_F = 0$$

We move the last part of each of these to the other side, and divide the first condition by the last. We get

$$\frac{n_H c_H^{-1/\sigma}}{n_F c_{FH}^{-1/\sigma}} = \frac{n_H p_H}{n_F \tau p_F}$$

$$\frac{c_H}{c_{FH}} = \left(\frac{p_H}{\tau p_F}\right)^{-\sigma} \tag{16}$$

Solving the budget constraint for c_{FH} and inserting above gives

$$\frac{c_H}{\frac{\mu w - n_H p_H c_H}{n_F \tau p_F}} = \left(\frac{p_H}{\tau p_F}\right)^{-\sigma}$$

$$c_H = \frac{{p_H}^{-\sigma}}{n_H {p_H}^{1-\sigma} + n_F (\tau p_F)^{1-\sigma}} \mu w$$

Moving back to (16) and instead inserting for c_H from the budget we can find c_{FH}

$$c_{FH} = \frac{(\tau p_F)^{-\sigma}}{n_H {p_H}^{1-\sigma} + n_F (\tau p_F)^{1-\sigma}} \mu w$$

Aggregated for the home country, we get the demand functions

$$C_H = \frac{{p_H}^{-\sigma}}{n_H {p_H}^{1-\sigma} + n_F (\tau p_F)^{1-\sigma}} \mu w L_H \tag{17}$$

$$C_{FH} = \frac{(\tau p_F)^{-\sigma}}{n_H {p_H}^{1-\sigma} + n_F (\tau p_F)^{1-\sigma}} \mu w L_H \tag{18}$$

Using the same method for the Foreign country, we get

$$C_F = \frac{{p_F}^{-\sigma}}{n_H (\tau p_H)^{1-\sigma} + n_F {p_F}^{1-\sigma}} \mu w L_F \tag{19}$$

$$C_{HF} = \frac{(\tau p_H)^{-\sigma}}{n_H (\tau p_H)^{1-\sigma} + n_F {p_F}^{1-\sigma}} \mu w L_F \tag{20}$$

The overall demand an H-good is $D_H = C_H + \tau C_{HF}$ (21)

The overall demand an F-good is $D_F = C_F + \tau C_{FH}$ (22)

If the number of firms is so large that the individual firm do not regard themselves as affecting the price indices, they will see themselves as facing a combined export and domestic demand curve with constant elasticity σ.

$$p = \frac{\sigma}{\sigma - 1}\beta w$$

Furthermore, as prices, wages and all other parameters relevant for the entry decision of firms (zero profit condition) are equal, we have an equalisation of firm output.

We turn to market clearing. We insert from (17) and (20) into (21) and from (18) and (19) into (22):

$$\frac{x}{\mu} = \frac{p^{-\sigma}}{n_H p^{1-\sigma} + n_F(\tau p)^{1-\sigma}} wL_H + \frac{(\tau p)^{-\sigma}}{n_F p^{1-\sigma} + n_H(\tau p)^{1-\sigma}} wL_F\tau \tag{23}$$

$$\frac{x}{\mu} = \frac{p^{-\sigma}}{n_F p^{1-\sigma} + n_H(\tau p)^{1-\sigma}} wL_F + \frac{(\tau p)^{-\sigma}}{n_H p^{1-\sigma} + n_F(\tau p)^{1-\sigma}} wL_H\tau \tag{24}$$

We simplify by choosing units $p = w = 1$ and by defining $\rho = \tau^{1-\sigma} < 1$

$$\frac{x}{\mu} = \frac{1}{n_H + n_F\rho} L_H + \frac{\rho}{n_F + n_H\rho} L_F \tag{25}$$

$$\frac{x}{\mu} = \frac{1}{n_F + n_H\rho} L_F + \frac{\rho}{n_H + n_F\rho} L_H \tag{26}$$

We want to solve these two equations for n_H and n_F.

We have three possible outcomes: 1) H produces all differentiated goods 2) F produces all differentiated goods 3) both countries produces some

We know consumers spend a constant share of income on the differentiated good. We also know that the zero-profit condition ensures all income in the differentiated sector goes to cover costs. With one factor of production these costs are all wage-costs. As wages are the same in the sectors, we conclude that a constant share of the total labour force $\mu(L_H + L_F)$ will be employed in the differentiated sector. From this we can find the number of varieties.

1) $$n_F = 0\,, \qquad n_H = \frac{\mu(L_H + L_F)}{x} \qquad (27.1)$$

2) $$n_H = 0\,, \qquad n_F = \frac{\mu(L_H + L_F)}{x} \qquad (27.2)$$

3) If both n_H and n_F are positive, we can solve (25) and (26) to get[2]:

$$\mathrm{s_n} = \frac{1}{(1-\rho)}[\mathrm{s_L} - \rho(1-\mathrm{s_L})] = \frac{1}{(1-\rho)}[(1+\rho)\mathrm{s_L} - \rho] \qquad (27.3)$$

where $\mathrm{s_L} = \frac{L_H}{L_H + L_F}$.

It makes sense to restrict $\mathrm{s_n}$ to be between zero and one. If the value of the expression above is outside of this, we will have full specialisation and be in either case 1) or 2). We can see that the limits for imperfect specialisation (case 3) are:

$$\frac{\rho}{1-\rho} < \mathrm{s_L} < \frac{1}{1-\rho}$$

This means that if the countries are sufficiently different in size, the largest country will produce all the differentiated goods. This is a version of the home market effect, that the larger country will produce more than a proportional share of the increasing returns good. This is not the same as the alternative formulation that countries tend to export what they have a relatively large demand of. But they both spring out of a more fundamental idea that increasing-returns industries, other things equal, will tend to locate where their market is larger.

[2] This is shown in the appendix *Solving for* $\mathrm{s_n}$.

Note that the width of the band of non-specialisation depends on ρ, which depends on τ. If transport costs are low, the band is narrow. As τ tends to one, we get full specialisation even if one country is only marginally larger. I consider it a problem with the fundamental framework that as transport costs move towards zero, the outcome has no similarity to the case with zero transport costs, which would imply no specialisation at all. This critique is similar to Thomas Holmes' and John Stevens' (2002) concern that as fixed costs move towards zero, the model bears no resemblance to the constant returns case.

4. Introducing heterogeneous productivity

In this chapter I present the model of Chaney that I use a a basis of my analysis. I go through the setup and his results, and derive everything from beginning to the end. I do change some things in his model, and the effects of this change compared to Chaney's will be noted. I will also draw comparisons to Krugman's model several times.

Utility of the consumer is given by:

$$U = q_0^{1-\mu}\left(\int_\Omega q(\omega)^{(\sigma-1)/\sigma}d\omega\right)^{[\sigma/(\sigma-1)]\mu}, \sigma > 1 \qquad (28)$$

As opposed to Chaney I have only one differentiated sector. Not sectors differentiated along substitutability. This does not affect results. We can still experiment with changing the value of sigma later on.

The homogeneous good 0 is freely traded and is used as a numeraire. It is produced under CRS with one unit of labour in country *n* producing w_n units of the good. I assume every country produces some of good 0, implying wages measured in the numeraire is proportional to w_n in country *n*.

Chaney uses variable and a fixed trade costs. Variable cost is of the iceberg type: If one unit of the differentiated good is shipped from country *i* to country *j*, only a fraction $1/\tau_{ij}$ arrives. In addition, if a firm from from country *i* exports to country *j*, it must pay a fixed cost f_{ij}. As opposed to Chaney, I look at a two-country case. Some of the calculations will be a lot more tedious in this case, as you cannot, as Chaney, ignore the impact a single country has on the totality.

Each firm draws a random productivity φ. The cost of producing q units of a good and selling them in country *j* for a firm with productivity φ is:

$$c_{ij}(q) = \frac{w_i\tau_{ij}}{\varphi}q + f_{ij} \qquad (29)$$

Firms are price setters. The optimal price charged in country *j* for a firm with productivity φ (after this called firm φ) is: $p_{ij}(\varphi) = \sigma/(\sigma-1)w_i\tau_{ij}/\varphi$. The productivity shocks are drawn from a Pareto distribution with shape parameter γ: productivity is distributed over [1,∞) according to

$$P(\tilde{\varphi} < \varphi) = G(\varphi) = 1 - \varphi^{-\gamma}, \gamma > \sigma - 1 \quad (30)$$

γ is an inverse measure of heterogeneity. (High γ - more homogeneous) Chaney assumes $\gamma > \sigma - 1$. The economic interpretation of this assumption is above my head.

Mass of potential entrants is proportional to the quantity of effective labour: $w_n L_n$. This means that even in the absence of economies of scale, we have imperfect competition and positive profits. Chaney does not assume free entry. Treating number of firms as purely exogenous simplifies the analysis, but may be considered dissatisfactorily if this is to be thought of as a long run model.

Each worker owns w_n shares of a global fund which collects all profits in the world.

4.1 Demand for the Differentiated goods

Income in country *n* is $Y_j = (1+\pi)\, w_j L_j$

Given the optimal pricing of firms and the demand by consumers, exports from country *i* to country *j, by firm φ are*

$$x_{ij}(\varphi) = p_{ij}(\varphi) q_{ij}(\varphi) = \mu Y_j \left(\frac{p_{ij}(\varphi)}{P_j}\right)^{1-\sigma} \quad (31)$$

where P_j is the price index for the differentiated good in country *j.* If only those firms above the productivity threshold $\bar{\varphi}_{ij}$ in country *i* export to country *j*, the ideal price index for the differentiated good in country *j*, P_j , and dividends per share , π, are defined as

$$P_j = \left(\sum_{i=1}^{2} w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \left(\frac{\sigma}{\sigma - 1} \frac{w_i \tau_{ij}}{\varphi} \right)^{1-\sigma} dG(\varphi) \right)^{1/(1-\sigma)} \tag{32}$$

$$\pi = \frac{\sum_{i,l=1}^{2} w_i L_i \left(\int_{\bar{\varphi}_{ij}}^{\infty} \pi_{ij}(\varphi) d\, G(\varphi) \right)}{\sum_{n=1}^{N} w_n L_n} \tag{33}$$

For the two country case, where $\pi_{ij}(\varphi) = \left(p_{ij}(\varphi) - c_{ij}(\varphi) \right) q_{ij}(\varphi) - f_{ij}$ are the net profits that a firm with productivity φ in country *i* earns from exporting to country *j*.

4.2 Trade with Heterogenous Firms

To compute global equilibrium of this world economy, I solve for the selection of firms into different export markets and generate predictions for aggregate bilateral trade flows.

The derivations below can be found in *Appendix: Chaney section II*

Productivity threshold: Exporters are only the most efficient of domestic firms.

Profits for firm φ exporting from country *i* in country *j*:

$$\pi_{ij}(\varphi) = \frac{\mu Y_j}{\sigma} \frac{\left[\sigma/(\sigma - 1) w_i \tau_{ij} / \varphi \right]^{1-\sigma}}{P_j^{1-\sigma}} - f_{ij}$$

We find the threshold $\bar{\varphi}_{ij}$ for exporting firms from $\pi_{ij}(\varphi) = 0$:

$$\bar{\varphi}_{ij} = \lambda_1 \left(\frac{f_{ij}}{Y_j} \right)^{1/(\sigma-1)} \frac{w_i \tau_{ij}}{P_j} \tag{34}$$

where

$$\lambda_1 = \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)} \frac{\sigma}{(\sigma-1)}$$

Equilibrium Price Indices for differentiated good: We start with (32):

$$P_j = \left(\sum_{k=1}^{2} w_k L_k \int_{\bar{\varphi}_{kj}}^{\infty} \left(\frac{\sigma}{\sigma-1}\frac{w_k \tau_{kj}}{\varphi}\right)^{1-\sigma} dG(\varphi)\right)^{1/(1-\sigma)}$$

and derive

$$P_j = \lambda_2 \times Y_j^{1/\gamma - 1/(\sigma-1)} \times \theta_j \qquad (35)$$

Where $\lambda_2 = \left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)-1/\gamma} \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1+\pi}{Y}\right)^{\frac{1}{\gamma}}$

and $\theta_j \equiv \left[\sum_{k=1}^{2} \frac{Y_k}{Y}\left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}}$

π is dividend per share and Y is world output. θ_j is a weighted average of bilateral trade barriers to *j*.

Equilibrium Exports, Thresholds and Profits: To find exports I insert from (35) and for p_{ij} into (31):

$$x_{ij}(\varphi) = \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1}, if\ \varphi > \bar{\varphi}_{ij} \qquad (36)$$

$$\lambda_3 = \sigma \lambda_4{}^{1-\sigma}, \qquad \lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{(1+\pi)}\right)\right]^{\frac{1}{\gamma}}$$

To find productivity threshold I enter (35) into (34):

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times f_{ij}^{\frac{1}{\sigma-1}} \qquad (37)$$

$$\lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{1+\pi}\right)\right]^{\frac{1}{\gamma}}$$

Aggregate output in country *i*, is defined by:

$$Y_i = (1+\pi)w_i L_i$$

Dividends per share can be found as

$$\pi = \frac{\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}{1-\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}$$

In summary:

$$x_{ij}\left(\varphi|\varphi > \bar{\varphi}_{ij}\right) = \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1} \tag{36}$$

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}} \tag{37}$$

$$Y_i = (1+\pi)w_i L_i \tag{38}$$

$$\pi = \frac{\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}{1-\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}} \tag{39}$$

$$\lambda_3 = \sigma \lambda_4{}^{1-\sigma}, \qquad \lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{(1+\pi)}\right)\right]^{\frac{1}{\gamma}}$$

$$\theta_j \equiv \left[\sum_{k=1}^{2} \frac{Y_k}{Y}\left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}}$$

This system can be solved from the bottom and up. (39) is only a function of exogenous variables. (38) is only a function of exogenous variables and π. (36) and (37) are only functions of exogenous variables, π and Y_i.

Aggregate Trade: We can find (see *appendix: Chanes section II*) an expression for aggregate trade in differentiated goods from country *i* to country *j* given as:

Proposition 1: Total exports (free on board) X_{ij} from country *i* to country *j* are given by:

$$X_{ij} = \mu \times \frac{Y_i Y_j}{Y} \left(\frac{w_i \tau_{ij}}{\theta_j} \right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)} \tag{40}$$

We call this the end of this chapter. We have derived the expressions we need. The utilisation of these expression will come in the next chapters.

5. Intensive versus Extensive Margins of trade

Using the equation in (10), we cannot differentiate between intensive and extensive margin. We therefore use the expression

$$X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi) \qquad (41)$$

Total differentiating this with respect to trade costs yields, using the Leibniz integral rule with infinity as the upper limit of integration gives

$$dX_{ij} = \left(w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} dG(\varphi) \right) d\tau_{ij} - \left(w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}} \right) d\tau_{ij} + \left(w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial f_{ij}} dG(\varphi) \right) df_{ij} - \left(w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial f_{ij}} \right) df_{ij} \qquad (42)$$

--------intensive margin-------- --------extensive margin----------

Following a reduction of trade barriers, each existing exporter $(\varphi > \bar{\varphi}_{ij})$ exports more. This is the intensive margin. At the same time, higher potential profits attract new entrants ($\bar{\varphi}_{ij}$ goes down). This is the extensive margin.

5.1 Chaney's n-country case

We denote the elasticity of trade with respect to variable trade costs ζ.

We denote the elasticity of trade with respect to fixed trade costs ξ.

We find see the components of ζ easiest from the expression:

$$X_{ij} = \gamma w_i L_i \lambda_3 \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \gamma \left(-\frac{1}{(\sigma-1-\gamma)}\right) \left[\lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}}\right]^{\sigma-1-\gamma} \qquad (43)$$

(an intermediate step from the appendix)

The first exponent of tau is the intensive margin, the last is the extensive margin.

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = -\frac{d\ln X_{ij}}{d\ln \tau_{ij}} = \quad (\sigma - 1) \quad + \quad \big(\gamma - (\sigma - 1)\big) \quad = \gamma \tag{44}$$

We can see that a large sigma leads to a large intensive margin-elasticity (which is what is anticipated by Krugman) while it dampens the extensive margin-elasticity with the same amount. Thus

$$\frac{\partial \zeta}{\partial \sigma} = 0 \tag{45}$$

To compare this to the Krugman case, we start with equation (20) and multiply with $\tau_{ij} p_i$ to find the value of exports. Remember $p_i = p_j = p$

$$X_{ij} = \tau_{ij} p C_{ij} = \frac{\left(\tau_{ij} p\right)^{1-\sigma}}{n_i \left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}} \mu w L_j$$

The elasticity in Krugmans two-country case, is complicated, and will be derived later. What Chaney does is that he compares with an n-country version of Krugman, which means that τ_{ij} is considered not to have an impact on the price index in the denominator. If we for now think of the denominator as constant, we get:

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau/\tau} = \sigma - 1$$

$$\frac{\partial \zeta}{\partial \sigma} > 0 \tag{46}$$

Elasticity of exports with respect to variable trade barriers in the Chaney model (γ) is larger than in the absence of heterogeneity, and larger than for each individual firm (both $\sigma - 1$). This is because a reduction in trade cost not only causes an increase in the exports of individual firms, but in addition allows new, less efficient firms to enter.

We use (43) to find ξ. As the fixed costs do not affect the intensive margin, the net effect equals the extensive effect.

$$\xi = -\frac{dX_{ij}/X_{ij}}{df_{ij}/f_{ij}} = -\frac{d\ln X_{ij}}{d\ln f_{ij}} = \quad 0 \quad + \quad \frac{\gamma}{\sigma-1} - 1 \tag{47}$$

We can see that

$$\frac{\partial\xi}{\partial\sigma} < 0 \tag{48}$$

The sum of the effects of the elasticity of substitution through fixed and variable trade costs is negative. Krugman gets the opposite effect because he has no fixed trade costs and no extensive margin (no heterogenous firms). As σ approaches $\gamma + 1$ from below, ξ approaches zero.

PROPOSITION 2 (Intensive and extensive margins): *The elasticity of substitution (σ) has no effect on the elasticity of trade flows with respect to variable trade costs (ζ), and a negative effect on the elasticity of trade flows with respect to fixed costs (ξ).*

5.2 My Two-country case

The above is true only if there are enough countries so that θ_j is not affected by a change in τ_{ij}. This may hold in Chaney's analysis, but not in the two country case. Chaney argues in a technical appendix on the internet that the two-country case only strengthens his findings, and I will show that this in a way is correct.

In the following I will therefore also take account of a change in θ_j.

$$\theta_j \equiv \left[\frac{Y_i}{Y}\left(w_i\tau_{ij}\right)^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + \frac{Y_j}{Y} f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}} \tag{49}$$

With this expression I do not know how to calculate the elasticities using logarithms. For simplicity, we use the expression for X_{ij} from (40):

$$X_{ij} = \mu \times \frac{Y_i Y_j}{Y}\left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

5.2.1 Elasticity of trade flows with respect to variable trade costs

We start from (40):

$$-\zeta = \frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = \frac{\mu \times \frac{Y_i Y_j}{Y} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}(-\gamma)\left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma-1} \frac{w_i \theta_j - w_i \tau_{ij}\frac{\partial \theta_j}{\partial \tau_{ij}}}{\theta_j{}^2}}{\mu \times \frac{Y_i Y_j}{Y}\left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}} \times \tau_{ij}$$

Simplifying:

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = \gamma\left(1 - \frac{\partial \theta_j}{\partial \tau_{ij}}\frac{\tau_{ij}}{\theta_j}\right) \qquad (50)$$

Setting $w_j = 1, \tau_{jj} = 1,$ we find from (49)

$$\frac{\partial \theta_j}{\partial \tau_{ij}} = -\frac{1}{\gamma}\left[\frac{Y_i}{Y}(w_i \tau_{ij})^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + \frac{Y_j}{Y} f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}-1} \times (-\gamma)\frac{Y_i}{Y}(w_i \tau_{ij})^{-\gamma-1} w_i f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$$

and multiply by $\frac{\tau_{ij}}{\theta_j}$ and simplifying:

$$\frac{\partial \theta_j}{\partial \tau_{ij}}\frac{\tau_{ij}}{\theta_j} = \frac{Y_i (w_i \tau_{ij})^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}{Y_i (w_i \tau_{ij})^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + Y_j f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}$$

We insert into (50) and simplify

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = \left(\frac{1}{1 + \frac{Y_i}{Y_j}\left(w_i \tau_{ij}\right)^{-\gamma}\left(\frac{f_{ij}}{f_{jj}}\right)^{\left(1-\frac{\gamma}{\sigma-1}\right)}} \right)$$

From this we can find:

$$\frac{\partial \zeta}{\partial \sigma} < 0 \qquad (51)$$

To compare this to Krugman's two-country case, we must derive the elasticity from (20)

$$X_{ij} = \tau_{ij} p C_{ij} = \frac{\left(\tau_{ij} p\right)^{1-\sigma}}{n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}} \mu w L_j$$

$$\frac{dX_{ij}}{d\tau_{ij}} = \frac{(1-\sigma)\left(\tau_{ij} p\right)^{-\sigma} \mu w L_j \left[p\left(n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}\right) - \left(\tau_{ij} p\right)^{1-\sigma} n_i p\right]}{\left(n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}\right)^2}$$

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = -\frac{(1-\sigma)\left[n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma} - \left(\tau_{ij} p\right)^{1-\sigma} n_i\right]}{n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}}$$

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = (\sigma - 1)\frac{n_j p^{1-\sigma}}{n_i\left(\tau_{ij} p\right)^{1-\sigma} + n_j p^{1-\sigma}}$$

$$\zeta = -\frac{dX_{ij}/X_{ij}}{d\tau_{ij}/\tau_{ij}} = (\sigma - 1)\frac{1}{\frac{n_i}{n_j}{\tau_{ij}}^{1-\sigma} + 1}$$

$$\frac{\partial \zeta}{\partial \sigma} > 0 \qquad (52)$$

So to sum up. In the n-country case, as can be seen from (45) and (46), Chaney gets no relation between σ and ζ, while Krugman anticipates a negative relation. As

Chaney states in his footnote, the result is in a way strengthened in the two-country case. The two-country Chaney model anticipates a positive relation while the two-country Krugman model again predicts a negative relation.

Even though Chaney is right that this is strictly stronger, it shows that $\frac{\partial \zeta}{\partial \sigma} = 0$ only is true for the special case where one country is too small to affect the world/trade region.

5.2.2 Elasticity of trade flows with respect to fixed trade costs

The other elasticity to explore, is the elasticity of trade with respect to fixed trade costs. Again we start from (40)

$$\xi = -\frac{dX_{ij}/X_{ij}}{df_{ij}/f_{ij}}$$

$$\xi = \frac{-\mu \times \frac{Y_i Y_j}{Y}\left[-\gamma\left(\frac{w_i\tau_{ij}}{\theta_j}\right)^{-\gamma-1}\left(-\frac{w_i\tau_{ij}}{\theta_j{}^2}\frac{\partial\theta_j}{\partial f_{ij}}\right)f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)} + \left(\frac{w_i\tau_{ij}}{\theta_j}\right)^{-\gamma}\left(1-\frac{\gamma}{\sigma-1}\right)f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}\right)}\right]f_{ij}}{\mu \times \frac{Y_i Y_j}{Y}\left(\frac{w_i\tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}}$$

$$\xi = -\left[\left(1-\frac{\gamma}{\sigma-1}\right) + \gamma\frac{\partial\theta_j}{\partial f_{ij}}\frac{f_{ij}}{\theta_j}\right] \tag{53}$$

We find from (17), after simplification

$$\frac{\partial\theta_j}{\partial f_{ij}}\frac{f_{ij}}{\theta_j} = -\frac{1}{\gamma}\frac{Y_i\left(w_i\tau_{ij}\right)^{-\gamma}\left(1-\frac{\gamma}{\sigma-1}\right)f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}{Y_i\left(w_i\tau_{ij}\right)^{-\gamma}f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + Y_j f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}$$

Inserting into (21) and rearranging :

$$\xi = -\left[\left(1-\frac{\gamma}{\sigma-1}\right) - \frac{Y_i\left(w_i\tau_{ij}\right)^{-\gamma}\left(1-\frac{\gamma}{\sigma-1}\right)f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}{Y_i\left(w_i\tau_{ij}\right)^{-\gamma}f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + Y_j f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}}\right]$$

$$\xi = \left(\frac{\gamma}{\sigma - 1} - 1\right)\left[\frac{1}{1 + \frac{Y_i}{Y_j}\left(w_i \tau_{ij}\right)^{-\gamma}\left(\frac{f_{ij}}{f_{jj}}\right)^{\left(1 - \frac{\gamma}{\sigma - 1}\right)}}\right]$$

The first factor is equal to Chaney's elasticity. The second factor in the square brackets is positive, but less than one. So this elasticity is weakened in the two-country case. The sigma present in these brackets draw in the same direction as the first, so sigma's impact on the elasticity ξ is strengthened in the two-country case.

$$\frac{\partial \xi}{\partial \sigma} < 0$$

Thus I have shown that Chaney's results do not come into existence because he leaves the two-country case. On the countrary, both his findings are strengthened when we take the model back to two countries where comparison with the Krugman model is better.

6. The Hunt for the Home Market Effect

Krugman, when writing about HME, writes that the large countries share of world production must be larger than the share of world income/size. So there is HME if the largest country has a larger than proportional share of the production. This implies that we have HME if

$$\frac{X_i}{X_i + X_j} > \frac{w_i L_i}{w_i L_i + w_j L_j}$$

for $w_i L_i > w_j L_j$, where total exports[3] X_i are given by

$$X_i = \frac{X_{ii}}{X_{ji} + X_{ii}} \mu(1+\pi) w_i L_i + \frac{X_{ij}}{X_{jj} + X_{ij}} \mu(1+\pi) w_j L_j \ , \qquad i, j = H, F$$

In Helpman/Krugman (1985) this is analysed with the number of varieties directly instead of the value of exports to each market. We cannot use their framework here for two reasons: we have a continuum of varieties and thus not a well-defined number of firms. Secondly, output varies between firms because of heterogeneous productivity. Therefore the number of firms is of no interest anyway.

Our first pursuit is to investigate how the shares of production $\frac{X_i}{X_i+X_j}$ depend on relative market size $\frac{\mathrm{Y_i}}{\mathrm{Y_j}} = \frac{w_i L_i}{w_j L_j}$.

We normalise $L_F = 1$ and $w_F = 1$.

We derive total share of production:

$$\mathrm{X_H} = \frac{\mathrm{X_{HF}}}{\mathrm{X_{HF}} + \mathrm{X_{FF}}} \mu(1+\pi) + \frac{\mathrm{X_{HH}}}{\mathrm{X_{HH}} + \mathrm{X_{FH}}} \mu(1+\pi) w_H L_H$$

[3] What I tend to term ”exports” are total ”exports both to the domestic market and the other market. It is really the value of total production of differentiated goods.

$$\mathrm{X_F} = \frac{\mathrm{X_{FF}}}{\mathrm{X_{FF}} + \mathrm{X_{HF}}}\mu(1+\pi) + \frac{\mathrm{X_{FH}}}{\mathrm{X_{FH}} + \mathrm{X_{HH}}}\mu(1+\pi) w_H L_H$$

$$\frac{\mathrm{X_H}}{\mathrm{X_F}} = \frac{\dfrac{\mathrm{X_{HF}}}{\mathrm{X_{HF}} + \mathrm{X_{FF}}} + \dfrac{\mathrm{X_{HH}}}{\mathrm{X_{HH}} + \mathrm{X_{FH}}} w_H L_H}{\dfrac{\mathrm{X_{FF}}}{\mathrm{X_{FF}} + \mathrm{X_{HF}}} + \dfrac{\mathrm{X_{FH}}}{\mathrm{X_{FH}} + \mathrm{X_{HH}}} w_H L_H}$$

Defining country H's market shares as U and V

$$U = \frac{\mathrm{X_{HF}}}{\mathrm{X_{HF}} + \mathrm{X_{FF}}} = \frac{\dfrac{\mathrm{X_{HF}}}{\mathrm{X_{FF}}}}{\dfrac{\mathrm{X_{HF}}}{\mathrm{X_{FF}}} + 1} = , \qquad \mathrm{V} = \frac{\mathrm{X_{HH}}}{\mathrm{X_{HH}} + \mathrm{X_{FH}}} = \frac{\dfrac{\mathrm{X_{HH}}}{\mathrm{X_{FH}}}}{\dfrac{\mathrm{X_{HH}}}{\mathrm{X_{FH}}} + 1}$$

$$\frac{\mathrm{X_H}}{\mathrm{X_F}} = \frac{\mathrm{U} + \mathrm{V} w_H L_H}{(1-\mathrm{U}) + (1-\mathrm{V}) w_H L_H}$$

$$\frac{\mathrm{X_F}}{\mathrm{X_H}} = \frac{1 - \mathrm{U} + w_H L_H - \mathrm{V} w_H L_H}{\mathrm{U} + \mathrm{V} w_H L_H} = \frac{1 + w_H L_H}{\mathrm{U} + \mathrm{V} w_H L_H} - 1$$

$$\frac{\mathrm{X_F} + \mathrm{X_H}}{\mathrm{X_H}} = \frac{1 + w_H L_H}{\mathrm{U} + \mathrm{V} w_H L_H}$$

$$\frac{\mathrm{X_H}}{\mathrm{X_H} + \mathrm{X_F}} = \frac{\mathrm{U} + \mathrm{V} w_H L_H}{1 + w_H L_H} = \mathrm{U}\frac{1}{1 + w_H L_H} + \mathrm{V}\frac{w_H L_H}{1 + w_H L_H}$$

Defining $\mathrm{S_X} = \frac{\mathrm{X_H}}{\mathrm{X_H} + \mathrm{X_F}}$, $\mathrm{S_L} = \frac{w_H L_H}{w_H L_H + 1}$ gives us

$$\mathrm{S_X} = \frac{\mathrm{X_H}}{\mathrm{X_H} + \mathrm{X_F}} = \mathrm{U}(1 - \mathrm{S_L}) + \mathrm{V}\mathrm{S_L}$$

$$\mathrm{S_X} = \frac{\mathrm{X_H}}{\mathrm{X_H} + \mathrm{X_F}} = \mathrm{U} + (\mathrm{V} - \mathrm{U})\mathrm{S_L} \tag{54}$$

which is a close parallel to Helpman/Krugman's (27). There are however vital differences. In the H/K's equation, $\mathrm{S_X}$ will equal zero for a strictly positive $\mathrm{S_L}$. We can rule that out in our equation. To get that, we would have to have $U > V$ which would imply that each country would have a larger market share abroad than at home.

That is clearly not likely. On the contrary, S_X tends to U as S_L tends to zero. The next thing we observe is that as S_L tends to one, S_X tends to V.

The next important difference is that U and V are not constants! They are themselves functions of relative size. We are therefore nowhere near disclosing the HME properties of this model. We can see that as long as both countries have a positive market share in both markets, $U > 0$, $V < 1$. We can see that complete specialisation can only take place if market shares can reach zero for $0 < S_L < 1$. This will be investigated next.

To sum up this first part: An increase in relative size will shift the importance of the markets. As the proportionality factor is higher in the domestic market than in the foreign market, each country will gain ground relatively if their own country grows.

6.1 The Total effect on National market shares

There may be four channels for the increase in production that comes from an increase in country size. We have so far studied the first. The last three comes from the determination of aggregate exports to each country X_{ij} . It is these aggregate exports to each market that must be analysed to be inserted into U and V above. Only when we know the behaviour of the market shares as relative size changes, we can say anything about the home market effect in Chaney's model.

Before we decompose exports into its constituent parts, let us first study the aggregate effect on market shares of a rise in relative size. From Chaney's *proposition 1* we have that total exports X_{ij} from country i to country j are given by:

$$X_{ij} = \mu \times \frac{Y_i Y_j}{Y} \left(\frac{w_i \tau_{ij}}{\theta_j} \right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

we normalise $L_j = 1$. For simplicity we also normalise $w_j = 1$ while τ_{jj} is already assumed equal to one.

$$X_{ij} = \mu \times \frac{(1+\pi)w_i L_i}{(w_i L_i + 1)} \left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

Relative production for market *j*.

$$\frac{X_{ij}}{X_{jj}} = \frac{\mu \times \frac{(1+\pi)w_i L_i}{(w_i L_i + 1)} \left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}}{\mu \times \frac{(1+\pi)}{(w_i L_i + 1)} \left(\frac{1}{\theta_j}\right)^{-\gamma} f_{jj}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}}$$

$$\frac{X_{ij}}{X_{jj}} = w_i{}^{1-\gamma} \tau_{ij}{}^{-\gamma} L_i \left(\frac{f_{ij}}{f_{jj}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)} \tag{55}$$

which is linearly increasing in L_i. If we consider market *i*.

$$\frac{X_{ii}}{X_{ji}} = \frac{\mu \times \frac{(1+\pi)w_i L_i}{(w_i L_i + 1)} \left(\frac{w_i \tau_{ii}}{\theta_i}\right)^{-\gamma} f_{ii}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}}{\mu \times \frac{(1+\pi)w_j L_j}{(w_i L_i + 1)} \left(\frac{w_j \tau_{ji}}{\theta_i}\right)^{-\gamma} f_{ji}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}}$$

$$\frac{X_{ii}}{X_{ji}} = w_i{}^{1-\gamma} \tau_{ji}{}^{\gamma} L_i \left(\frac{f_{ii}}{f_{ji}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)} \tag{56}$$

which is also increasing in L_i. The sum of this is that as the relative size of country *i* increases, its market share increases as well. Note that as relative size tends to zero, relative supply tends to zero. As relative size tends to infinity, relative production tends to infinity. We can see from (55) and (56) that *relative* sales are proportional to *relative* size. The factors of proportionality are here

$$w_i{}^{1-\gamma} \tau_{ij}{}^{-\gamma} \left(\frac{f_{ij}}{f_{jj}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

$$w_i{}^{1-\gamma} \tau_{ji}{}^{\gamma} \left(\frac{f_{ii}}{f_{ji}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

The factor of proportionality can be larger than, equal to or smaller than one. Only if it is equal to one, the share of production will also be proportional to the share of world size. In this case it cannot be equal to one in both markets. Therefore the share of production will generally be different from share of size. It is not given, however, that the largest country will have a larger than proportional share in each market. On the contrary, the country with the most fortunate parameter values will supply a larger than proportional share independent of size. Note that the transformation from relative value to shares is not difficult. Market share is a function of relative size:

$$\frac{\mathrm{X_{HF}}}{\mathrm{X_{HF}} + \mathrm{X_{FF}}} = \frac{\frac{\mathrm{X_{HF}}}{\mathrm{X_{FF}}}}{\frac{\mathrm{X_{HF}}}{\mathrm{X_{FF}}} + 1}$$

U and *V* can therefore be considered either as the share or as a function of relative values.

6.2 Decomposing national market shares

We have studied the first component of the share of total *world* production, namely the relative size of the *national* markets. The last three all add up to national market shares $\frac{\mathrm{X_{ij}}}{\mathrm{X_{ij}} + \mathrm{X_{jj}}}$. We now decompose into the determinants of national market shares. We will study this in terms of relative sizes, and translate this into shares later.

Inspecting this expression for aggregate exports from country *i* to country *j*

$$X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi) \qquad (41)$$

we can see that if L_i increases, we will have a proportional increase in the density of firms ($w_i L_i$). Dividing sales from *i* to *j* with *j*'s domestic sales:

$$\frac{X_{ij}}{X_{jj}} = w_i L_i \frac{\int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{\int_{\bar{\varphi}_{jj}}^{\infty} x_{jj}(\varphi) dG(\varphi)}$$

we see that the direct effect of firm density on relative sales are linear in relative size L_i. This factor may therefore explain all variation in relative production, as we have seen above from (55) and (56) that relative sales will indeed be linear in relative size. This means that we anticipate the integral to be constant as size changes. We will investigate this.

The two other possible channels of change are a change in the productivity cut-offs (limits of integration) and the value of output per firm $x_{ij}(\varphi)$. We will try to decompose exports' dependence on size in the three components. As the first (firm density) is trivial, we go to the cut-offs.

6.3 How the limit of entry varies with country size

We start from the expression for θ_j from (35) and equation (37). As I am not really that interested in the exogenous wage differences, I put $w_i = w_j = 1$. $\tau_{ii} = \tau_{jj}$ is already assumed equal to one. L_i is as above relative size.

$$\theta_j \equiv \left[\frac{Y_i}{Y} \tau_{ij}{}^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} + \frac{Y_j}{Y} \tau_{jj}{}^{-\gamma} f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}} \tag{35}$$

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{\tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}} \tag{37}$$

We start by inserting for the Ys and work with θ_j first.

For convenience, we use $\tau_{ij}{}^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} = A$ and $f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)} = B$

We also assume that it is more costly to get access to the foreign market than the home market, and that each market is most easily accessed from itself. This is a weak assumption: $f_{ij}\,,\ f_{ji} > f_{ii}\,,\ f_{jj}$

National income is found from (38)

$$Y_i = (1+\pi)\, L_i$$

$$\theta_j \equiv \left[\frac{(1+\pi)L_i}{(1+\pi)(L_i+1)}A + \frac{(1+\pi)}{(1+\pi)(L_i+1)}B\right]^{-\frac{1}{\gamma}}$$

$$\theta_j \equiv \left[\frac{L_i}{(L_i+1)}A + \frac{1}{(L_i+1)}B\right]^{-\frac{1}{\gamma}} \tag{57}$$

$$\frac{\partial\theta_j}{\partial L_i} = -\frac{1}{\gamma}\left[\frac{L_i}{(L_i+1)}A + \frac{1}{(L_i+1)}B\right]^{-\frac{1}{\gamma}-1}\left[\frac{1}{(L_i+1)^2}A - \frac{1}{(L_i+1)^2}B\right]$$

$$\frac{\partial\theta_j}{\partial L_i} = -\frac{1}{\gamma}\theta_j\,\frac{\frac{1}{(L_i+1)^2}A - \frac{1}{(L_i+1)^2}B}{\frac{L_i}{(L_i+1)}A + \frac{1}{(L_i+1)}B}$$

$$\frac{\partial\theta_j}{\partial L_i} = -\frac{1}{\gamma}\theta_j\,\frac{1}{(L_i+1)}\frac{(A-B)}{(L_iA+B)} \tag{58}$$

We go back to the limit of integration $\bar{\varphi}_{ij}$ inserted for Y and Y_j:

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{(1+\pi)(L_i+1)}{(1+\pi)}\right)^{\frac{1}{\gamma}} \times \left(\frac{\tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}}$$

$$\bar{\varphi}_{ij} = \lambda_4 \times (L_i+1)^{\frac{1}{\gamma}} \times \left(\frac{\tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}}$$

$$\frac{\partial\bar{\varphi}_{ij}}{\partial L_i} = \lambda_4 \times f_{ij}{}^{\frac{1}{\sigma-1}}\left[\frac{1}{\gamma}(L_i+1)^{\frac{1}{\gamma}-1} \times \left(\frac{\tau_{ij}}{\theta_j}\right) + (L_i+1)^{\frac{1}{\gamma}} \times \left(-\frac{\tau_{ij}}{\theta_j{}^2}\right)\frac{\partial\theta_j}{\partial L_i}\right]$$

$$= \lambda_4 (L_i + 1)^{\frac{1}{\gamma}} \left(\frac{\tau_{ij}}{\theta_j}\right) f_{ij}{}^{\frac{1}{\sigma-1}} \left[\frac{1}{\gamma}(L_i + 1)^{-1} - \frac{1}{\theta_j}\frac{\partial\theta_j}{\partial L_i}\right]$$

$$= \bar{\varphi}_{ij} \left[\frac{1}{\gamma}(L_i + 1)^{-1} - \frac{1}{\theta_j}\frac{\partial\theta_j}{\partial L_i}\right]$$

$$\frac{\partial\bar{\varphi}_{ij}}{\partial L_i} = \bar{\varphi}_{ij} \left[\left(\frac{1}{\gamma(L_i + 1)}\right) - \frac{1}{\theta_j}\frac{\partial\theta_j}{\partial L_i}\right]$$

Inserting for $\frac{\partial\theta_j}{\partial L_i}$ from (58) gives

$$= \bar{\varphi}_{ij} \left[\left(\frac{1}{\gamma(L_i + 1)}\right) - \frac{1}{\theta_j}\left(-\theta_j \frac{1}{\gamma(L_i + 1)}\frac{(A - B)}{(L_i A + B)}\right)\right]$$

$$\frac{\partial\bar{\varphi}_{ij}}{\partial L_i} = \bar{\varphi}_{ij} \frac{1}{\gamma(L_i + 1)}\left[1 + \frac{A - B}{L_i A + B}\right] \qquad (59)$$

We can see that this is positive for all values of the parameters. The change is also proportional to the limit itself.

The conclusion is

$$\frac{\partial\bar{\varphi}_{ij}}{\partial L_i} > 0$$

This was the first step in studying the limits of entry. We have three more limits of integration to consider: $\bar{\varphi}_{jj}$, $\bar{\varphi}_{ji}$ and $\bar{\varphi}_{ii}$.

The next limit I consider is the cut-off for firms in *j* to access its domestic market. The limit, inserted for Y and Y_j is given by

$$\bar{\varphi}_{jj} = \lambda_4 \times (L_i + 1)^{\frac{1}{\gamma}} \times \frac{1}{\theta_j} f_{jj}{}^{\frac{1}{\sigma-1}}$$

θ_j is same as before, and calculation the same as above. We get

$$\frac{\partial\bar{\varphi}_{jj}}{\partial L_i} = \bar{\varphi}_{jj}\,\frac{1}{\gamma(L_i+1)}\left[1+\frac{A-B}{L_iA+B}\right] \tag{60}$$

which is positive for all positive values of the parameters. As above, the change is proportional to the limit itself, and interestingly, the proportionality factor is the same.

Therefore:

$$\frac{\partial\bar{\varphi}_{jj}}{\partial L_i} > 0$$

For the last two limits we need $\frac{\partial\theta_i}{\partial L_i}$.

$$\theta_i \equiv \left[\frac{L_i}{(L_i+1)}C + \frac{1}{(L_i+1)}D\right]^{-\frac{1}{\gamma}} \tag{61}$$

$$\frac{\partial\theta_i}{\partial L_i} = -\theta_i\,\frac{1}{\gamma(L_i+1)}\frac{C-D}{(L_iC+D)} \tag{62}$$

where $C = f_{ii}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$and $D = \tau_{ji}{}^{-\gamma} f_{ji}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$

We then calculate the cut-off for entry from *i* to its domestic market

$$\bar{\varphi}_{ii} \;= \lambda_4 \times \left(1+\frac{1}{L_i}\right)^{\frac{1}{\gamma}} \times \left(\frac{1}{\theta_i}\right) \times f_{ii}{}^{\frac{1}{\sigma-1}}$$

$$\frac{\partial\bar{\varphi}_{ii}}{\partial L_i} = \lambda_4 \times f_{ii}{}^{\frac{1}{\sigma-1}}\left[\frac{1}{\gamma}\left(1+\frac{1}{L_i}\right)^{\frac{1}{\gamma}-1}\left(-\frac{1}{(L_i)^2}\right)\times\left(\frac{1}{\theta_i}\right)+\left(1+\frac{1}{L_i}\right)^{\frac{1}{\gamma}}\left(-\frac{1}{\theta_i{}^2}\frac{\partial\theta_i}{\partial L_i}\right)\right]$$

$$\frac{\partial\bar{\varphi}_{ii}}{\partial L_i} = -\bar{\varphi}_{ii}\left[\frac{1}{\gamma(L_i+1)}\left(\frac{1}{L_i}\right)+\frac{1}{\theta_i}\frac{\partial\theta_i}{\partial L_i}\right]$$

Inserting:

$$\frac{\partial\bar{\varphi}_{ii}}{\partial L_i} = -\bar{\varphi}_{ii}\,\frac{1}{\gamma(L_i+1)}\left[\frac{1}{L_i}-\frac{C-D}{L_iC+D}\right] \tag{63}$$

We can see that $\frac{1}{L_i} \geq \frac{C-D}{L_i C+D}$ for all parameter values, so the expression is negative.

The last limit:

$$\bar{\varphi}_{ji} = \lambda_4 \times \left(1+\frac{1}{L_i}\right)^{\frac{1}{\gamma}} \times \left(\frac{\tau_{ji}}{\theta_i}\right) \times f_{ji}{}^{\frac{1}{\sigma-1}}$$

$$\frac{\partial \bar{\varphi}_{ji}}{\partial L_i} = -\bar{\varphi}_{ji} \frac{1}{\gamma(L_i+1)}\left[\frac{1}{L_i} - \frac{C-D}{L_i C+D}\right] \tag{64}$$

As above, we can see that this is necessarily negative. We have that the changes in the limits are proportional to the limits themselves, and with the same factor of proportionality between them, but different from those for access to country *j*.

We collect the four derivatives from above:

$$\frac{\partial \bar{\varphi}_{ij}}{\partial L_i} = \bar{\varphi}_{ij} \frac{1}{\gamma(L_i+1)}\left[1+\frac{A-B}{L_i A+B}\right] > 0 \tag{59}$$

$$\frac{\partial \bar{\varphi}_{jj}}{\partial L_i} = \bar{\varphi}_{jj} \frac{1}{\gamma(L_i+1)}\left[1+\frac{A-B}{L_i A+B}\right] > 0 \tag{60}$$

$$\frac{\partial \bar{\varphi}_{ii}}{\partial L_i} = -\bar{\varphi}_{ii} \frac{1}{\gamma(L_i+1)}\left[\frac{1}{L_i} - \frac{C-D}{L_i C+D}\right] < 0 \tag{63}$$

$$\frac{\partial \bar{\varphi}_{ji}}{\partial L_i} = -\bar{\varphi}_{ji} \frac{1}{\gamma(L_i+1)}\left[\frac{1}{L_i} - \frac{C-D}{L_i C+D}\right] < 0 \tag{64}$$

$$A = \tau_{ij}{}^{-\gamma} f_{ij}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)},\ B = f_{jj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)},\ C = f_{ii}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)},\ D = \tau_{ji}{}^{-\gamma} f_{ji}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$$

Thus both countries will sell a broader band of varieties in country *i* if this country grows relative to *j*. So the growing market will receive goods from increasingly ineffective firms in both countries while the cut-offs into the relatively diminishing country will rise.

6.4 Relative Output per Variety

At last, we investigate if a change in relative size has an effect on relative sales per variety. This is the third determinant of relative national market shares. We know that

$$x_{ij}(\varphi) = p_{ij}(\varphi)q_{ij}(\varphi) = \mu Y_j \left(\frac{p_{ij}(\varphi)}{P_j}\right)^{1-\sigma} = \mu Y_j \left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi}\tau_{ij}}{P_j}\right)^{1-\sigma}$$

and from this we find

$$\frac{x_{ij}(\varphi)}{x_{jj}(\varphi)} = \tau_{ij}{}^{1-\sigma}, \qquad \frac{x_{ii}(\varphi)}{x_{ji}(\varphi)} = \left(\frac{1}{\tau_{ji}}\right)^{1-\sigma}$$

As we can see, the relative sales (into each market) of varieties produced with the same productivity level are not dependent on size. Relative sales of each variety therefore add nothing to the aggregate effect of size.

6.5 Total Effect on Share of production

To sum up this section, we bring back (54), (55), (56) and the expressions for U and V.

$$S_X = \frac{X_i}{X_i + X_j} = U + (V - U)S_L \tag{54}$$

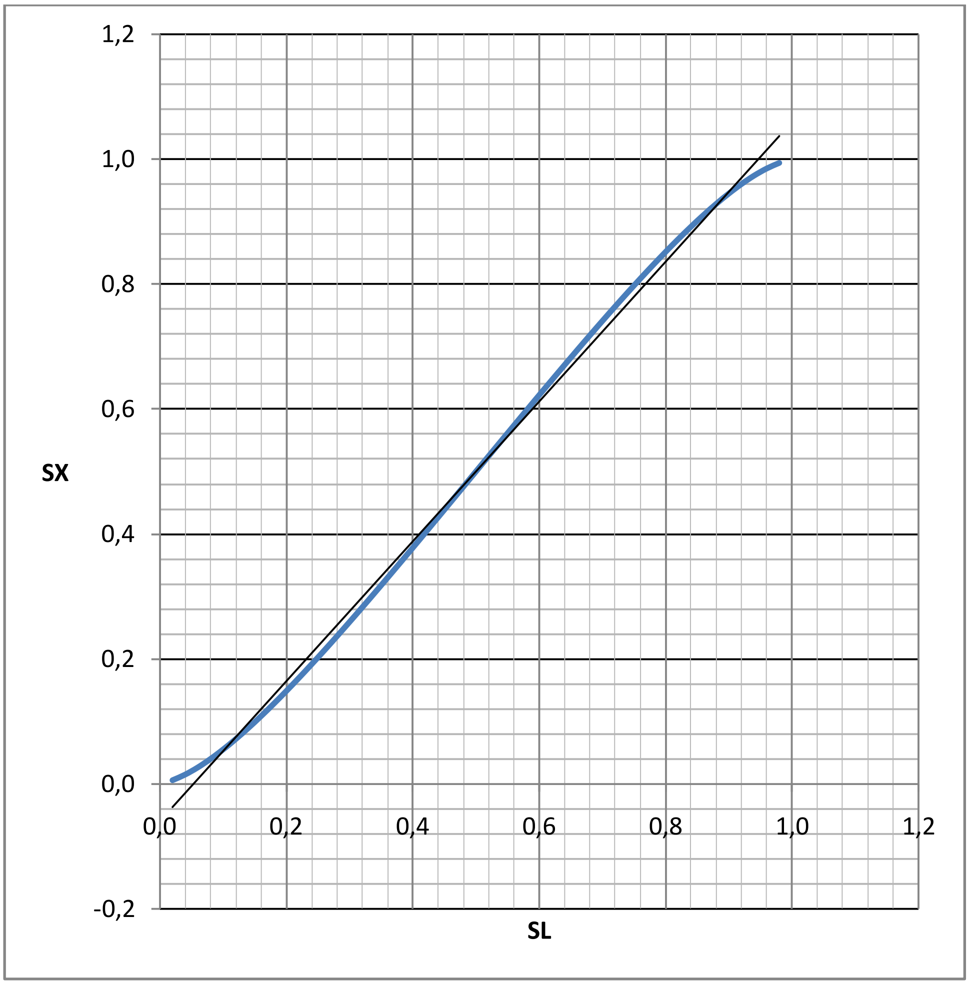
$$U = \frac{\frac{X_{ij}}{X_{jj}}}{\frac{X_{ij}}{X_{jj}} + 1}, \qquad V = \frac{\frac{X_{ii}}{X_{ji}}}{\frac{X_{ii}}{X_{ji}} + 1}$$

$$\frac{X_{ij}}{X_{jj}} = \tau_{ij}{}^{-\gamma} L_i \left(\frac{f_{ij}}{f_{jj}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)} \tag{55}$$

$$\frac{X_{ii}}{X_{ji}} = \tau_{ji}{}^{\gamma} L_i \left(\frac{f_{ii}}{f_{ji}}\right)^{-\left(\frac{\gamma}{\sigma-1}-1\right)} \tag{56}$$

We can see from these expressions that as S_L tends to zero, S_X tends to U. But when S_L tends to zero L_i also tends to zero. That implies that U tends to zero as well. So the total effect is, unsurprisingly, that country i will not have positive output when empty of workers.

As S_L tends to one L_i tends to infinity and S_X tends to V. But again; V is not constant but tends to one. So we know now that S_X as a function of S_L crosses through the origin and the point (1,1). We turn to a figure to get an idea of the shape.



We can see that contrary to the Helpman/Krugman model, this model have no intercepts with $S_X = 0,\ S_X = 1$. We also see that it has a nonlinear shape. That means that for some intervals, a rise in S_L leads to a less than proportional rise in S_X, while on the middle interval the rise in S_X is larger than proportional. If we study the best fitted linear approximation, we can see that this resembles the Helpman/Krugman-figure. This is indeed evidence of a HME! If a 45º line from the origin is imagined above, it would reveal that for $S_L > 1/2 \rightarrow S_X > S_L$.

Manipulating the parameters does not give me a stronger non-linear shape than the above. The curve tends to lie very close to the 45º line, so the home market effect is not strong.

Finally, I will add some intuition to the technicalities. The most important question is why this figure gets a different shape from the Helpman/Krugman-figure. I believe the chief reason for never having complete specialisation is that no matter how small a country grows, there will always be a few extremely productive firms able to overcome any trade obstacles and size disadvantages. This model (like H/K) does not allow firms to move to the largest country. They are stuck where they are, and can only choose production or non-production.

But as far as I can understand, there is also another reason that adds to this. The system of fixed costs makes it viable to produce for the home market even if a firm is too inefficient to enter the foreign market. This effect will not work when S_L is very large or very small however, as the limit of entry will actually be lower into the large foreign country than into the small domestic country for great size differences.

Thus I have shown that Chaney's model preserve the HME, if only weakly. The model does not predict full specialisation, which is contrary to Helpman/Krugman.

7. Impact of a Change in Size on the Price Indices

The ideal price index is a measure of the minimum cost of obtaining one unit of utility. Given that the models we are looking at have exogenous wage formation income is not affected by trade. Therefore the ideal price index is an inverse measure of utility. A lower price index leads to a higher utility level.

I would like to see how the price indices is affected by a change in country size. For comparison, I will first go through how price indices are effected by a change in country size in the standard Krugman framework, with and without a homogeneous good. Then I go to the Chaney model.

One note. As I normalise $L_F = 1$ this will of course affect the level of the price index. That might be considered unfortunate, but is not a large problem. Although I tend to write about *relative size*, I believe that might be inaccurate. It is not the same to study a rise in L_F and a fall in L_H unless we only study the price indices relative to each other. A rise in L_H can in the below therefore *not* be understood as a fall in L_F.

7.1 The impact of a change in size on price indices – Krugman case

7.1.1 Without a Homogeneous good

Two country case with symmetric trade costs and no homogeneous good we get:

$$P_i = \left[\frac{L_i}{\sigma\alpha}p^{1-\sigma} + \frac{L_j}{\sigma\alpha}(\tau p)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad i,j = H,F$$

$$\frac{\partial P_i}{\partial L_i} = \frac{1}{1-\sigma}\left[\frac{L_i}{\sigma\alpha}p^{1-\sigma} + \frac{L_j}{\sigma\alpha}(\tau p)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}}\frac{1}{\sigma\alpha}p^{1-\sigma}$$

$$\frac{\partial P_i}{\partial L_j} = \frac{1}{1-\sigma}\left[\frac{L_i}{\sigma\alpha}p^{1-\sigma} + \frac{L_j}{\sigma\alpha}(\tau p)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}}\frac{1}{\sigma\alpha}(\tau p)^{1-\sigma}$$

These are both negative (as $\sigma > 1$). We can also see that $\frac{\partial P_i}{\partial L_i} / \frac{\partial P_i}{\partial L_j} = \frac{1}{\tau^{1-\sigma}} = \tau^{\sigma-1} > 1$, so the price index of the growing country will have its price index most affected.

7.1.2 Introducing a Homogeneous good

Note that in the coming, this price index is an index of differentiated goods only. I am uncertain as to whether including the homogeneous good will change the qualitative results. The expression for the price index is taken from the denominator in (18).

$$P_i = \left[n_i p^{1-\sigma} + n_j (\tau p)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad i,j = H,F \tag{65}$$

$$\frac{\partial P_i}{\partial L_H} = \frac{1}{1-\sigma}\left[n_i p^{1-\sigma} + n_j (\tau p)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} \times p^{1-\sigma}\left(\frac{\partial n_i}{\partial L_H} + \frac{\partial n_j}{\partial L_H}\tau^{1-\sigma}\right), \qquad i,j = H,F \tag{66}$$

We find values for n_i and n_j from (27) and translate it into *H,F* notation with $L_F = 1$. We also insert for $s_L = \frac{L_H}{L_H + L_F} = \frac{1}{1+\frac{1}{L_H}}$

$$n_H = s_n \frac{\mu(L_H+1)}{x} = \frac{1}{(1-\rho)}\left[(1+\rho)s_L - \rho\right]\frac{\mu(L_H+1)}{x} = \left(\frac{1+\rho}{1+\frac{1}{L_H}} - \rho\right)\frac{\mu(L_H+1)}{x(1-\rho)}$$

$$n_F = (1-s_n)\frac{\mu(L_H+1)}{x} = \left(\frac{1-(1+\rho)s_L}{(1-\rho)}\right)\frac{\mu(L_H+1)}{x} = \left(1 - \frac{1+\rho}{1+\frac{1}{L_H}}\right)\frac{\mu(L_H+1)}{x(1-\rho)}$$

$$\frac{\partial n_H}{\partial L_H} = \frac{\mu}{x(1-\rho)} > 0 \tag{67.1}$$

$$\frac{\partial n_F}{\partial L_H} = -\frac{\mu\rho}{x(1-\rho)} < 0 \tag{67.2}$$

Inserting into (66)we get

$$\frac{\partial P_H}{\partial L_H} = \frac{1}{1-\sigma}\left[n_H p^{1-\sigma} + n_F (\tau p)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} \times p^{1-\sigma}\frac{\mu}{x(1-\rho)}(1-\rho\tau^{1-\sigma})$$

$$\frac{\partial P_F}{\partial L_H} = \frac{1}{1-\sigma}[n_F p^{1-\sigma} + n_H(\tau p)^{1-\sigma}]^{\frac{\sigma}{1-\sigma}} \times p^{1-\sigma}\frac{\mu}{x(1-\rho)}(\tau^{1-\sigma} - \rho)$$

It is the last term that will govern the signs here. We can see that

$(1 - \rho\tau^{1-\sigma}) > 0$, thus $\frac{\partial P_H}{\partial L_H} < 0$

$(\tau^{1-\sigma} - \rho) = 0$, thus $\frac{\partial P_F}{\partial L_H} = 0$

So from the Krugman case we can derive the following results:

- A country will be better off if it is large.
- A country is indifferent between whether the trading partner is small or large.

I think this last result is strange, as it seems to rule out gains from trade. My math seems to be correct, so my guess is that this is only true as long as the countries are not too dissimilar in size, so that the homogeneous sector can absorb spare capacity and ensure wage equalisation. We now turn to the Chaney model.

7.2 The impact of a change in size on price indices – Chaney case

My question now is what sort of results Chaney's model will yield. From an expression for the price index (intermediate step in *appendix: Chaney section II*) we have by setting $L_F = w_F = 1$

$$P_j = \frac{\sigma}{\sigma-1}\left(\gamma\sum_{k=i}^{j} w_k L_k \left(w_k \tau_{kj}\right)^{1-\sigma}\int_{\bar{\varphi}_{kj}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi\right)^{1/(1-\sigma)} \tag{68}$$

$$P_H = \frac{\sigma}{\sigma-1}\left[\gamma L_H {w_H}^{2-\sigma}\int_{\bar{\varphi}_{HH}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi + \gamma(\tau_{FH})^{1-\sigma}\int_{\bar{\varphi}_{FH}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi\right]^{1/(1-\sigma)} \tag{69.1}$$

$$P_F = \frac{\sigma}{\sigma - 1}\left[\gamma L_H {w_H}^{2-\sigma} {\tau_{HF}}^{1-\sigma} \int_{\bar{\varphi}_{HF}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi + \gamma \int_{\bar{\varphi}_{FF}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi\right]^{1/(1-\sigma)} \qquad (69.2)$$

We differentiate these with respect to L_H, using the Leibniz integral rule with infinity as the upper limit of integration.

$$\frac{\partial P_H}{\partial L_H} = \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1}{1-\sigma}\right) [\dots]_H{}^{\frac{\sigma}{1-\sigma}} \times \frac{\partial [\dots]_H}{\partial L_H} \qquad (70.1)$$

$$\frac{\partial [\dots]_H}{\partial L_H} = \gamma {w_H}^{2-\sigma} \int_{\bar{\varphi}_{HH}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi - \gamma L_H {w_H}^{2-\sigma} \left({\bar{\varphi}_{HH}}{}^{\sigma-2-\gamma} \frac{\partial \bar{\varphi}_{HH}}{\partial L_H}\right)$$
$$- \gamma (\tau_{FH})^{1-\sigma} \left({\bar{\varphi}_{FH}}{}^{\sigma-2-\gamma} \frac{\partial \bar{\varphi}_{FH}}{\partial L_H}\right) \qquad (71.1)$$

The effect of a change in L_H on the index for country F can be found as

$$\frac{\partial P_F}{\partial L_H} = \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1}{1-\sigma}\right) [\dots]_F{}^{\frac{\sigma}{1-\sigma}} \times \frac{\partial [\dots]_F}{\partial L_H} \qquad (70.2)$$

$$\frac{\partial [\dots]_F}{\partial L_H} = \gamma {w_H}^{2-\sigma} {\tau_{HF}}^{1-\sigma} \int_{\bar{\varphi}_{HF}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi - \gamma L_H {w_H}^{2-\sigma} {\tau_{HF}}^{1-\sigma} \left({\bar{\varphi}_{HF}}{}^{\sigma-2-\gamma} \frac{\partial \bar{\varphi}_{HF}}{\partial L_H}\right)$$
$$- \gamma \left({\bar{\varphi}_{FF}}{}^{\sigma-2-\gamma} \frac{\partial \bar{\varphi}_{FF}}{\partial L_H}\right) \qquad (71.2)$$

We have already found the derivatives of the cut-offs as (59), (60), (63) and (64). The result of the integration has been found in the Chaney-section.

$$\int_{\bar{\varphi}_{ij}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi = \left(\frac{1}{\gamma - (\sigma - 1)}\right) {\bar{\varphi}_{ij}}{}^{\sigma-1-\gamma} \qquad (72)$$

We can insert from (72) and for all the limits into (71) and then from (71) into (70). There is not much point in doing that literally, since the resultant expression does not simplify much. It is better to analyse the results from the equations above.

7.3 Analysis

I can predict with certainty the signs of the φ's and their derivatives. Before I go further, I collect all these equations, and assume $w_i = 1$ and symmetric trade costs

$$f_{ij} = f_{ji} = f_{for} \text{ , } f_{ii} = f_{jj} = f_{dom} \text{ , } \tau_{ij} = \tau_{ji} = \tau$$

$$\bar{\varphi}_{ij} = \lambda_4 \times (L_i + 1)^{\frac{1}{\gamma}} \times \left(\frac{\tau}{\theta_j}\right) f_{for}{}^{\frac{1}{\sigma-1}}$$

$$\bar{\varphi}_{jj} = \lambda_4 \times (L_i + 1)^{\frac{1}{\gamma}} \times \left(\frac{1}{\theta_j}\right) f_{dom}{}^{\frac{1}{\sigma-1}}$$

$$\bar{\varphi}_{ii} = \lambda_4 \times \left(1 + \frac{1}{L_i}\right)^{\frac{1}{\gamma}} \times \left(\frac{1}{\theta_i}\right) f_{dom}{}^{\frac{1}{\sigma-1}}$$

$$\bar{\varphi}_{ji} = \lambda_4 \times \left(1 + \frac{1}{L_i}\right)^{\frac{1}{\gamma}} \times \left(\frac{\tau}{\theta_i}\right) f_{for}{}^{\frac{1}{\sigma-1}}$$

From the above we can see that $\bar{\varphi}_{ij} = \tau \left(\frac{f_{for}}{f_{dom}}\right)^{\frac{1}{\sigma-1}} \bar{\varphi}_{jj}$ and $\bar{\varphi}_{ji} = \tau \left(\frac{f_{for}}{f_{dom}}\right)^{\frac{1}{\sigma-1}} \bar{\varphi}_{ii}$. We use our results to analyse the derivatives. With symmetric costs the structure simplifies and we have

$B = C = f_{dom}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$ and $A = D = \tau^{-\gamma} f_{for}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}$. I insert for C and D

$$\frac{\partial \bar{\varphi}_{ij}}{\partial L_i} = \bar{\varphi}_{ij} \frac{1}{\gamma(L_i + 1)} \left[1 + \frac{A - B}{L_i A + B}\right] > 0 \tag{59}$$

$$\frac{\partial \bar{\varphi}_{jj}}{\partial L_i} = \bar{\varphi}_{jj} \frac{1}{\gamma(L_i + 1)} \left[1 + \frac{A - B}{L_i A + B}\right] > 0 \tag{60}$$

$$\frac{\partial \bar{\varphi}_{ii}}{\partial L_i} = -\bar{\varphi}_{ii} \frac{1}{\gamma(L_i + 1)} \left[\frac{1}{L_i} - \frac{B - A}{L_i B + A}\right] < 0 \tag{63}$$

$$\frac{\partial \bar{\varphi}_{ji}}{\partial L_i} = -\bar{\varphi}_{ji} \frac{1}{\gamma(L_i + 1)} \left[\frac{1}{L_i} - \frac{B - A}{L_i B + A}\right] < 0 \tag{64}$$

We can see that $\frac{\partial\bar{\varphi}_{ij}}{\partial L_i} = \tau\left(\frac{f_{for}}{f_{dom}}\right)^{\frac{1}{\sigma-1}}\frac{\partial\bar{\varphi}_{jj}}{\partial L_i}$ and $\frac{\partial\bar{\varphi}_{ji}}{\partial L_i} = \tau\left(\frac{f_{for}}{f_{dom}}\right)^{\frac{1}{\sigma-1}}\frac{\partial\bar{\varphi}_{ii}}{\partial L_i}$. Below, I will insert this.

We take one step up and inspect the price indices in a reduced version of (69):

$$P_i = \frac{\sigma}{\sigma-1}\left[\gamma L_i\int_{\bar{\varphi}_{ii}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi + \gamma\tau^{1-\sigma}\int_{\bar{\varphi}_{ji}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi\right]^{1/(1-\sigma)}$$

$$P_j = \frac{\sigma}{\sigma-1}\left[\gamma L_i\tau^{1-\sigma}\int_{\bar{\varphi}_{ij}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi + \gamma\int_{\bar{\varphi}_{jj}}^{\infty}\varphi^{\sigma-2-\gamma}\,d\varphi\right]^{1/(1-\sigma)}$$

We are interested in the value in the brackets. In the case of symmetric costs, the largest country has the largest value in the brackets. We move to the derivatives from (70) inserted from (71) and rearrange:

$$\frac{\partial[\ldots]_i}{\partial L_i} = \gamma\bar{\varphi}_{ii}{}^{\sigma-1-\gamma}\left[\left(\frac{1}{\gamma-(\sigma-1)}\right) - L_i\left(\frac{1}{\bar{\varphi}_{ii}}\frac{\partial\bar{\varphi}_{ii}}{\partial L_i}\right)\right] - \gamma\tau^{1-\sigma}\left(\bar{\varphi}_{ji}{}^{\sigma-2-\gamma}\frac{\partial\bar{\varphi}_{ji}}{\partial L_i}\right)$$

$$\frac{\partial[\ldots]_j}{\partial L_i} = \gamma\bar{\varphi}_{ij}{}^{\sigma-1-\gamma}\tau^{1-\sigma}\left[\left(\frac{1}{\gamma-(\sigma-1)}\right) - L_i\left(\frac{1}{\bar{\varphi}_{ij}}\frac{\partial\bar{\varphi}_{ij}}{\partial L_i}\right)\right] - \gamma\left(\bar{\varphi}_{jj}{}^{\sigma-2-\gamma}\frac{\partial\bar{\varphi}_{jj}}{\partial L_i}\right)$$

I insert from the known relations between the limits and their derivatives, simplify and rearrange:

$$\frac{\partial[\ldots]_i}{\partial L_i} = \gamma\bar{\varphi}_{ii}{}^{\sigma-1-\gamma}\left[\left(\frac{1}{\gamma-(\sigma-1)}\right) - \frac{1}{\bar{\varphi}_{ii}}\frac{\partial\bar{\varphi}_{ii}}{\partial L_i}\left(L_i + \tau^{-\gamma}\left(\frac{f_{for}}{f_{dom}}\right)^{1-\frac{\gamma}{\sigma-1}}\right)\right]$$

$$\frac{\partial[\ldots]_j}{\partial L_i} = \gamma\tau^{-\gamma}\left(\frac{f_{for}}{f_{dom}}\right)^{1-\frac{\gamma}{\sigma-1}}\bar{\varphi}_{jj}{}^{\sigma-1-\gamma}\left[\left(\frac{1}{\gamma-(\sigma-1)}\right) - \frac{1}{\bar{\varphi}_{jj}}\frac{\partial\bar{\varphi}_{jj}}{\partial L_i}\left(L_i + \tau^{\gamma}\left(\frac{f_{for}}{f_{dom}}\right)^{\frac{\gamma}{\sigma-1}-1}\right)\right]$$

Inserting for the derivatives, inserting A and B for trade costs and rearranging:

$$\frac{\partial[\ldots]_i}{\partial L_i} = \gamma\bar{\varphi}_{ii}{}^{\sigma-1-\gamma}\left[\left(\frac{1}{\gamma-(\sigma-1)}\right) + \frac{1}{\gamma(L_i+1)}\left[\frac{1}{L_i} - \frac{B-A}{L_iB+A}\right]\left(L_i + \frac{A}{B}\right)\right]$$

$$\frac{\partial[\dots]_j}{\partial L_i} = \gamma \frac{A}{B} \bar{\varphi}_{jj}{}^{\sigma-1-\gamma} \left[\left(\frac{1}{\gamma - (\sigma-1)} \right) - \frac{1}{\gamma(L_i+1)} \left[1 + \frac{A-B}{L_i A + B} \right] \left(L_i + \frac{B}{A} \right) \right]$$

We can from direct inspection of the above say for sure that $\frac{\partial[\dots]_i}{\partial L_i} > 1$.

I am not able to get any further algebraically. I am not sure what are reasonable values for the fixed costs. As we have normalised wages to unity, the fixed costs can be measured in labour units. As we also have normalised labour in country j to unity, fixed costs should be less than one. That means that what is larger of A and B depends on many parameters. Not that knowing that would make us able to say anything certain about $\frac{\partial[\dots]_j}{\partial L_i}$.

I see no other solution than using numerical tests. No matter how hard I try, I am not able to produce a negative result. I am also not able to produce $\frac{\partial[\dots]_i}{\partial L_i} < \frac{\partial[\dots]_j}{\partial L_i}$.

Although I have not proven it, I take it for a somewhat sketchy result. Now we bring back (70). As we said above: for symmetric costs, the larger country will have the larger value inside the parentheses.

$$\frac{\partial P_H}{\partial L_H} = \left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{1-\sigma} \right) [\dots]_H{}^{\frac{\sigma}{1-\sigma}} \times \frac{\partial[\dots]_H}{\partial L_H} \tag{70.1}$$

$$\frac{\partial P_F}{\partial L_H} = \left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{1-\sigma} \right) [\dots]_F{}^{\frac{\sigma}{1-\sigma}} \times \frac{\partial[\dots]_F}{\partial L_H} \tag{70.2}$$

The signs of this seem to be ok.

$$\frac{\partial P_H}{\partial L_H} < 0\,, \qquad \frac{\partial P_F}{\partial L_H} < 0$$

Thus everyone will be better off. In the Krugman model with homogeneous good we got the somewhat surprising result that a country is indifferent to the size of the country it trades with. In the Chaney model, we see that a country benefits from having a large trading partner.

Sadly, numerical testing does not give clear results on which index is the most affected. All we can say is that if country H is smaller, equal or not too much greater than country F, we will have

$$\left|\frac{\partial P_H}{\partial L_H}\right| > \left|\frac{\partial P_F}{\partial L_H}\right|$$

This will be fulfilled even for quite large size differences if the values of the other parameters are right. I believe the reason for having the opposite result comes from that if $P_F \gg P_H$ then the absolute change in the price index will be bigger, while the percentage change will not. Numerical tests support this. At every try I get

$$\frac{\frac{\partial P_H}{\partial L_H}}{\frac{\partial P_F}{\partial L_H}} \times \frac{P_F}{P_H} > 1$$

My conclusion is that the outcomes of this model points in the same general direction as of the Krugman model. The results from the Chaney model actually have more in common with the Krugman model without the homogeneous good. As for the economic interpretation of why the results differ, specifically why $\frac{\partial P_F}{\partial L_H} = 0$ in Krugman, I am unable to produce a solid explanation.

8. Conclusions

I follow up Chaney's analysis of the gravity effect by transferring it to the two-country case. My findings are that his results holds also in this case. For the n-country case, the Krugman model implies that the elasticity of substitution (σ) affects the elasticity of trade flows with respect to variable trade costs (ς) positively. Chaney finds that this effect is absent in his model. On the contrary, the elasticity of trade flows with respect to fixed trade costs (ξ) is affected negatively by σ. Thus the sum of this is that trade flows are affected negatively by σ, not positively as Krugman's model implies.

In the two-country case, the results are stronger. Here the Krugman model implies a positive relationship between σ and ς while the Chaney model implies a negative. In addition, the effect on ξ is negative, and more so than in the n-country case.

My next results come from analysing the home market effect in the model. I find that the effect is present, but is graphically visible only for the right parameter values. It thus seems to be rather week. The result I obtain is a non-linear relationship between the share of labour force and the share of production. For the symmetric case, the smaller country will produce a somewhat less than proportional share of the differentiated good. The non-linear shape still ensures that we never get full specialisation. This is caused by the productivity structure: there will always be a few firms productive enough to overcome the disadvantages of being located in the smaller market.

Finally, I analyse the impact of a change in country size on the ideal price indices. As income per capita is constant, the impact on the price index can be seen as a proxy for the impact on utility. I derive how the gains from trade (as measured by the price index) depend on the size of the partner one trades with.

For the Krugman model I find that a country benefits from getting bigger. Somewhat surprisingly I find that it if a homogeneous good is also traded, it matters not for a country whether the trading partner is small or large. For the Chaney model the results are less clear, but a combination of algebra and numerical analysis using MATLAB indicates that a country gains from getting bigger, and that it also gains from having a larger trading partner. The results are not entirely clear on which country benefits the most in absolute terms, but it seems clear that the growing country will a larger percentagewise reduction in the price index. Paradoxically, the implications of the Chaney-model appear to have more in common with those derived from the Krugman-model without a homogeneous good, than those from the model with this good.

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Appendix

Gains from variety

Consumers in both countries will have access to the sum of varieties $n_H + n_F$. This obviously leads to gains from trade. This can be seen if we re-express (1) now that we know that in the absence of trade costs $c_i = c \ \ p_i = p$ for all *i*. Utility and the budget constraint (assuming it binding) can be expressed as:

$$u = nc_i^{(\sigma-1)/\sigma}, \qquad 1 < \sigma \tag{1'}$$

$$np_i c_i = w$$

where *n* stands for all available varieties, i.e. n_K in the autarky case, $n_H + n_F$ in the trade case. Solving the constraint for c_i and inserting into the utility function, we get

$$u = n\left(\frac{w}{np_i}\right)^{(\sigma-1)/\sigma} = n^{1/\sigma}\left(\frac{w}{p_i}\right)^{(\sigma-1)/\sigma}$$

$$\frac{\partial u}{\partial n} = \frac{1}{\sigma} n^{(1-\sigma)/\sigma}\left(\frac{w}{p_i}\right)^{(\sigma-1)/\sigma} > 0$$

while

$$\frac{\partial^2 u}{\partial n^2} = \frac{1-\sigma}{\sigma^2} n^{(1-2\sigma)/\sigma}\left(\frac{w}{p_i}\right)^{(\sigma-1)/\sigma} < 0$$

Marginal utility with respect to variety is positive and diminishing.

From (5) we find

$$\lambda = \frac{\sigma - 1}{\sigma p_i} c_i^{-1/\sigma}$$

As c_i is diminishing in *n*, marginal utility of income λ is increasing in *n*.

Solving for share of production S_{n}

$$\frac{x}{\mu}=\frac{1}{n_H+n_F\rho}L_H+\frac{\rho}{n_F+n_H\rho}L_F \tag{25}$$

$$\frac{x}{\mu}=\frac{1}{n_F+n_H\rho}L_F+\frac{\rho}{n_H+n_F\rho}L_H \tag{26}$$

We start from $(25)=(26)$

$$\frac{1}{n_H+n_F\rho}L_H+\frac{\rho}{n_F+n_H\rho}L_F=\frac{1}{n_F+n_H\rho}L_F+\frac{\rho}{n_H+n_F\rho}L_H$$

$$\frac{1-\rho}{n_H+n_F\rho}L_H=\frac{1-\rho}{n_F+n_H\rho}L_F$$

$$L_H(n_F+n_H\rho)=L_F(n_H+n_F\rho)$$

$$n_F(L_H-L_F\rho)=n_H(L_F-L_H\rho)$$

$$n_H=n_F\frac{(L_H-L_F\rho)}{(L_F-L_H\rho)}$$

We define

$$\mathrm{s_n}\equiv\frac{n_H}{n_H+n_F}=\frac{n_F\frac{(L_H-L_F\rho)}{(L_F-L_H\rho)}}{n_F\frac{(L_H-L_F\rho)}{(L_F-L_H\rho)}+n_F}=\frac{\frac{L_H-L_F\rho}{L_F-L_H\rho}}{\frac{L_H-L_F\rho}{L_F-L_H\rho}+\frac{L_F-L_H\rho}{L_F-L_H\rho}}$$

$$=\frac{L_H-L_F\rho}{L_H-L_F\rho+L_F-L_H\rho}=\frac{L_H-L_F\rho}{(1-\rho)(L_H+L_F)}=\frac{1}{(1-\rho)}\left[\frac{L_H}{L_H+L_F}-\rho\frac{L_F}{L_H+L_F}\right]$$

$$\mathrm{s_n}=\frac{1}{(1-\rho)}[\mathrm{s_L}-\rho(1-\mathrm{s_L})]=\frac{1}{(1-\rho)}[(1+\rho)\mathrm{s_L}-\rho] \tag{27}$$

where

$$s_L = \frac{L_H}{L_H + L_F}$$

Derivations from Chaney section II

Productivity threshold:

Exporters are only the most efficient of domestic firms.

Price of good φ from country i in country j:

$$p_{ij}(\varphi) = \sigma/(\sigma-1) w_i \tau_{ij}/\varphi$$

Quantity consumed of the same:

$$q_{ij}(\varphi) = \mu \frac{p_{ij}(\varphi)^{-\sigma}}{P_j^{1-\sigma}} Y_j$$

And comparing

$$\pi_{ij}(\varphi) = \left(p_{ij}(\varphi) - c_{ij}(\varphi)\right) q_{ij}(\varphi) - f_{ij}$$

with $\qquad c_{ij}(q) = \frac{w_i \tau_{ij}}{\varphi} q + f_{ij}$

we see that $\qquad c_{ij,\varphi}(q) = c_{ij}(\varphi) q_{ij}(\varphi) + f_{ij} = \frac{w_i \tau_{ij}}{\varphi} q_{ij}(\varphi) + f_{ij}$

and thus $c_{ij}(\varphi) = \frac{w_i \tau_{ij}}{\varphi}$

$$\pi_{ij}(\varphi) = \left(p_{ij}(\varphi) - \frac{w_i \tau_{ij}}{\varphi}\right) q_{ij}(\varphi) - f_{ij}$$

Inserting for $p_{ij}(\varphi)$ and $q_{ij}(\varphi)$:

$$\pi_{ij}(\varphi) = \left(\sigma/(\sigma-1)\frac{w_i \tau_{ij}}{\varphi} - \frac{w_i \tau_{ij}}{\varphi}\right) \mu \frac{\left[\sigma/(\sigma-1) w_i \tau_{ij}/\varphi\right]^{-\sigma}}{P_j^{1-\sigma}} Y_j - f_{ij}$$

$$\pi_{ij}(\varphi) = \mu Y_j (1 - (\sigma - 1)/\sigma) \frac{\left[\sigma/(\sigma-1) w_i \tau_{ij}/\varphi\right]^{1-\sigma}}{P_j^{1-\sigma}} - f_{ij}$$

Profits for firm φ exporting from country i in country j:

$$\pi_{ij}(\varphi) = \frac{\mu Y_j}{\sigma} \frac{\left[\sigma/(\sigma-1) w_i \tau_{ij}/\varphi\right]^{1-\sigma}}{P_j^{1-\sigma}} - f_{ij}$$

We find the threshold $\bar{\varphi}_{ij}$ for exporting firms from $\pi_{ij}(\varphi) = 0$:

$$\frac{\mu Y_j}{\sigma} \frac{\left[\sigma/(\sigma-1) w_i \tau_{ij}/\varphi\right]^{1-\sigma}}{P_j^{1-\sigma}} - f_{ij} = 0$$

$$\varphi = \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)} \frac{\sigma}{(\sigma-1)} \left(\frac{f_{ij}}{Y_j}\right)^{1/(\sigma-1)} \frac{w_i \tau_{ij}}{P_j}$$

$$\bar{\varphi}_{ij} = \lambda_1 \left(\frac{f_{ij}}{Y_j}\right)^{1/(\sigma-1)} \frac{w_i \tau_{ij}}{P_j} \qquad (34)$$

where

$$\lambda_1 = \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)} \frac{\sigma}{(\sigma-1)}$$

Equilibrium Price Indices for the differentiated good:

We start with (32):

$$P_j = \left(\sum_{k=1}^{2} w_k L_k \int_{\bar{\varphi}_{kj}}^{\infty} \left(\frac{\sigma}{\sigma-1} \frac{w_k \tau_{kj}}{\varphi}\right)^{1-\sigma} dG(\varphi)\right)^{1/(1-\sigma)}$$

From (30) we see that $\dfrac{dG(\varphi)}{d\varphi} = \dfrac{d(1-\varphi^{-\gamma})}{d\varphi} = \gamma\varphi^{-\gamma-1} \to dG(\varphi) = \gamma\varphi^{-\gamma-1}d\varphi$

$$P_j = \left(\sum_{k=1}^{2} w_k L_k \int_{\bar{\varphi}_{kj}}^{\infty} \left(\frac{\sigma}{\sigma-1}\frac{w_k \tau_{kj}}{\varphi}\right)^{1-\sigma} \gamma\varphi^{-\gamma-1}d\varphi\right)^{1/(1-\sigma)} \qquad k = 1,2 \quad , \qquad j = 1,2$$

We take out of the integrand everything that does not concern φ

$$P_j = \frac{\sigma}{\sigma-1}\left(\gamma \sum_{k=1}^{2} w_k L_k \left(w_k \tau_{kj}\right)^{1-\sigma} \int_{\bar{\varphi}_{kj}}^{\infty} \varphi^{\sigma-2-\gamma}\, d\varphi\right)^{1/(1-\sigma)} \tag{68}$$

$$P_j = \frac{\sigma}{\sigma-1}\left(\gamma \sum_{k=1}^{2} w_k L_k \left(w_k \tau_{kj}\right)^{1-\sigma} \Bigg|_{\bar{\varphi}_{ij}}^{\infty} \frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)}\right)^{1/(1-\sigma)}$$

If $\frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)} = F(\varphi)$ then

$$P_j = \frac{\sigma}{\sigma-1}\left(\gamma \sum_{k=1}^{2} w_k L_k \left(w_k \tau_{kj}\right)^{1-\sigma} \left[F(\infty) - F(\bar{\varphi}_{ij})\right]\right)^{1/(1-\sigma)}$$

As we have assumed $\gamma > \sigma - 1$ we get $\lim_{\varphi\to\infty} \frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)} = 0$

Therefore, inserting for $\bar{\varphi}_{ij}$ from (34) we get

$$P_j = \frac{\sigma}{\sigma-1}\left(\gamma \sum_{k=1}^{2} w_k L_k \left(w_k \tau_{kj}\right)^{1-\sigma} \left(-\frac{1}{(\sigma-1-\gamma)}\right)\left[\lambda_1 \left(\frac{f_{kj}}{Y_j}\right)^{1/(\sigma-1)} \frac{w_k \tau_{kj}}{P_j}\right]^{\sigma-1-\gamma}\right)^{1/(1-\sigma)}$$

Inserting for λ_1 and rearranging

$$P_j = \left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} \left(\frac{\sigma}{\sigma-1}\right) Y_j^{\frac{1}{\gamma}-\frac{1}{\sigma-1}} \left[\sum_{k=1}^{2} w_k L_k \left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}^{\frac{(\sigma-1-\gamma)}{(\sigma-1)}}\right]^{-\frac{1}{\gamma}}$$

We know that

$$Y_k = (1+\pi) w_k L_k \;\rightarrow\; w_k L_k = \frac{Y_k}{(1+\pi)}$$

Inserting and rearranging

$$P_j = \left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{1+\pi}{Y}\right)^{\frac{1}{\gamma}}$$
$$\times Y_j^{\frac{1}{\gamma}-\frac{1}{\sigma-1}} \left[\sum_{k=1}^{2} \frac{Y_k}{Y} \left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}}$$

$$P_j = \lambda_2 \times Y_j^{1/\gamma - 1/(\sigma-1)} \times \theta_j \tag{35}$$

Where $\lambda_2 = \left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)-1/\gamma} \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{1+\pi}{Y}\right)^{\frac{1}{\gamma}}$

and $\theta_j \equiv \left[\sum_{k=1}^{2} \frac{Y_k}{Y} \left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}}$

π is dividend per share and Y is world output. θ_j is a weighted average of bilateral trade barriers to *j*.

Equilibrium Exports, Thresholds and Profits:

To find exports I insert from (35) and for p_{ij} into (31):

$$x_{ij}(\varphi) = \mu Y_j \left(\frac{\sigma/(\sigma-1) w_i \tau_{ij}/\varphi}{\left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{1+\pi}{Y}\right)^{\frac{1}{\gamma}} Y_j^{\frac{1}{\gamma}-\frac{1}{\sigma-1}} \theta_j} \right)^{1-\sigma}$$

$$x_{ij}(\varphi) = \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1}, if\ \varphi > \bar{\varphi}_{ij}$$

$$\lambda_3 = \sigma {\lambda_4}^{1-\sigma}, \qquad \lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{(1+\pi)}\right)\right]^{\frac{1}{\gamma}}$$

To find productivity threshold I enter (35) into (34):

$$\bar{\varphi}_{ij} = \lambda_1 \left(\frac{f_{ij}}{Y_j}\right)^{\frac{1}{\sigma-1}} \frac{w_i \tau_{ij}}{\left(\frac{\gamma-(\sigma-1)}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}-\frac{1}{\gamma}} \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1+\pi}{Y}\right)^{\frac{1}{\gamma}} Y_j^{\frac{1}{\gamma}-\frac{1}{\sigma-1}} \theta_j}$$

After some tedious rearrangement:

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times {f_{ij}}^{\frac{1}{\sigma-1}}$$

$$\lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{1+\pi}\right)\right]^{\frac{1}{\gamma}}$$

Aggregate output in country *i*, is by definition:

$$Y_i \equiv (1+\pi) w_i L_i \tag{38}$$

The dividends per share can be found as[4].

$$\pi = \frac{\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}{1-\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}$$

In summary:

$$x_{ij}\left(\varphi | \varphi > \bar{\varphi}_{ij}\right) = \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1} \tag{36}$$

[4] To find this is a complecated procedure that can be found for instance in appendix 8.3 of the working paper ”*The Margins of Multinational Production and the Role of Intra-Firm Trade”* by Irarrazabal, Moxnes and Opromolla (2008). Unfortunately, I did not have time to transfer this derivation into the framework and the notation of the Chaney model.

$$\bar{\varphi}_{ij} = \lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}} \tag{37}$$

$$Y_i = (1+\pi) w_i L_i \tag{38}$$

$$\pi = \frac{\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}}{1-\left(\frac{\sigma-1}{\gamma}\right)\frac{\mu}{\sigma}} \tag{39}$$

$$\lambda_3 = \sigma \lambda_4{}^{1-\sigma}, \qquad \lambda_4 = \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{(1+\pi)}\right)\right]^{\frac{1}{\gamma}}$$

$$\theta_j \equiv \left[\sum_{k=1}^{2} \frac{Y_k}{Y} \left(w_k \tau_{kj}\right)^{-\gamma} f_{kj}{}^{\left(1-\frac{\gamma}{\sigma-1}\right)}\right]^{-\frac{1}{\gamma}}$$

Aggregate Trade:

To find an expression for aggregate trade in differentiated goods from country *i* to country *j*, I start with

$$X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi) \tag{41}$$

Inserting (36) we get

$$X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1} dG(\varphi)$$

From (30) we see that $\dfrac{dG(\varphi)}{d\varphi} = \dfrac{d(1-\varphi^{-\gamma})}{d\varphi} = \gamma \varphi^{-\gamma-1} \to dG(\varphi) = \gamma \varphi^{-\gamma-1} d\varphi$

$$X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \varphi^{\sigma-1} \gamma \varphi^{-\gamma-1} d\varphi$$

We take out of the integrand everything that does not concern φ

$$X_{ij} = \gamma w_i L_i \lambda_3 \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \int_{\bar{\varphi}_{ij}}^{\infty} \varphi^{\sigma-2-\gamma} d\varphi$$

$$X_{ij} = \gamma w_i L_i \lambda_3 \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \Bigg|_{\bar{\varphi}_{ij}}^{\infty} \frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)}$$

If $\frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)} = F(\varphi)$ then

$$X_{ij} = \gamma w_i L_i \lambda_3 \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \left[F(\infty) - F(\bar{\varphi}_{ij})\right]$$

As we have assumed $\gamma > \sigma - 1$ $we\ get$ $\lim_{\varphi \to \infty} \frac{\varphi^{\sigma-1-\gamma}}{(\sigma-1-\gamma)} = 0$

Therefore, inserting for $\bar{\varphi}_{ij}$ from (37) we get

$$X_{ij} = \gamma w_i L_i \lambda_3 \left(\frac{Y_j}{Y}\right)^{\frac{\sigma-1}{\gamma}} \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \left(-\frac{1}{(\sigma-1-\gamma)}\right) \left[\lambda_4 \times \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}} \times \left(\frac{w_i \tau_{ij}}{\theta_j}\right) \times f_{ij}{}^{\frac{1}{\sigma-1}}\right]^{\sigma-1-\gamma} \tag{43}$$

$$X_{ij} = \gamma \lambda_3 \lambda_4{}^{\sigma-1-\gamma} \frac{1}{\gamma-(\sigma-1)} \times \frac{w_i L_i Y_j}{Y} \left(\frac{w_i \tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}{}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

The constant term simplifies:

$$\gamma \lambda_3 \lambda_4{}^{\sigma-1-\gamma} \frac{1}{\gamma-(\sigma-1)} = \gamma \sigma \lambda_4{}^{1-\sigma} \lambda_4{}^{\sigma-1-\gamma} \frac{1}{\gamma-(\sigma-1)} = \gamma \sigma \lambda_4{}^{-\gamma} \frac{1}{\gamma-(\sigma-1)}$$

$$= \gamma\sigma\left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\left(\frac{1}{(1+\pi)}\right)\right]^{-1}\frac{1}{\gamma-(\sigma-1)}$$

$$= \gamma\sigma\left(\frac{\mu}{\sigma}\right)\left(\frac{\gamma-(\sigma-1)}{\gamma}\right)(1+\pi)\frac{1}{\gamma-(\sigma-1)}$$

$$= (1+\pi)\mu$$

We know that $Y_i = (1+\pi)w_iL_i$ so $(1+\pi)\mu = \frac{Y_i}{w_iL_i}\mu$

$$X_{ij} = \frac{Y_i}{w_iL_i}\mu \times \frac{w_iL_iY_j}{Y}\left(\frac{w_i\tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}^{\;-\left(\frac{\gamma}{\sigma-1}-1\right)}$$

We finally get

Proposition 1: Total exports (free on board) X_{ij} from country *i* to country *j* are given by:

$$X_{ij} = \mu \times \frac{Y_iY_j}{Y}\left(\frac{w_i\tau_{ij}}{\theta_j}\right)^{-\gamma} f_{ij}^{\;-\left(\frac{\gamma}{\sigma-1}-1\right)} \qquad (40)$$